

**Definition.** The *trigonometric basis of period*  $2L$  is  $\mathcal{T}_L = \{\cos(\pi n x / L), \sin(\pi n x / L), n = 0, 1, 2, \dots\} \setminus \{0\}$ .

**Example.** For  $2L = 2\pi$ , the (canonical) trigonometric basis is  $\{1, \cos x, \sin x, \cos 2x, \sin 2x, \dots\}$ .

We'll assume  $L = \pi$  henceforth.

**Definition.** A *trigonometric series* is a linear combination of the trigonometric basis functions

$$T = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx).$$

Many periodic functions  $f(x) = f(x + 2\pi)$  can be represented by Fourier series with coefficients:

$$a_0 = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(x) dx,$$

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \cos(nx) dx,$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(x) \sin(nx) dx,$$

known as the *Euler formulas*.