1 2

The *Euler formulas* for periodic functions with period $2L$, $f(x) = f(x + 2L)$ are

$$a_0 = \frac{1}{2L} \int_{-L}^{L} f(x) \, \mathrm{d}x,$$

$$a_n = \frac{1}{L} \int_{-L}^{L} f(x) \cos\left(\frac{n\pi x}{L}\right) \mathrm{d}x,$$

$$b_n = \frac{1}{L} \int_{-L}^{L} f(x) \sin\left(\frac{n\pi x}{L}\right) \mathrm{d}x.$$

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left( a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right)$$

Even, functions, $f(x) = f(-x)$ are represented by a Fourier cosine series

$$f(x) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi x}{L}\right)$$

Odd, functions, $f(x) = -f(-x)$ are represented by a Fourier sine series

$$f(x) = \sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi x}{L}\right)$$