Consider a truncation of the infinite series

$$f(x) = a_0 + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right)$$

that defines a *trigonometric polynomial* of degree N

$$F(x) = a_0 + \sum_{n=1}^{N} \left(a_n \cos\left(\frac{n\pi x}{L}\right) + b_n \sin\left(\frac{n\pi x}{L}\right) \right).$$

The *square* (or 2-norm) *error* is defined as

$$E = \int_{-\pi}^{\pi} [f(x) - F(x)]^2 \, \mathrm{d}x$$

For $a_n = A_n$, $b_n = B_n$

$$E^* = \int_{-\pi}^{\pi} [f(x)]^2 \, \mathrm{d}x - \pi \left[2a_0^2 + \sum_{n=1}^{N} (a_n^2 + b_n^2) \right]$$

In general

$$E \geqslant E^* \geqslant 0$$

$$E^* \geqslant 0 \Rightarrow \frac{1}{\pi} \int_{-\pi}^{\pi} [f(x)]^2 \, \mathrm{d}x \geqslant 2a_0^2 + \sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$

known as the Bessel inequality.