The wave equation is a second-order, linear partial differential equation (PDE)

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2} \Leftrightarrow u_{tt} - c^2 u_{xx} = 0$$

models propagation of waves in a one-dimensional medium, such as a guitar string.

- One approach is to seek solutions of the form $u(t,x) = y(x)\exp(i\omega t)$ leading to the second-order equation

$$y'' + k^2 y = 0, k^2 = \omega^2 / c^2$$

- Another approach is to seek a transformation of the independent variables that leads to a solvable PDE

$$\left(\frac{\partial^2}{\partial t^2} - c^2 \frac{\partial^2}{\partial x^2}\right) u = \left(\frac{\partial}{\partial t} + c\frac{\partial}{\partial x}\right)\left(\frac{\partial}{\partial t} - c\frac{\partial}{\partial x}\right) u = \frac{\partial^2 u}{\partial w \partial v} = u_{vw} = 0,$$

  with the transformation of independent variables defined as

$$v = x + ct, w = x - ct,$$

  leading to a solution of the form

$$u(t,x) = \phi(x+ct) + \psi(x-ct).$$

  Given the initial conditions $u(0,x) = f(x)$, $u_t(0,x) = g(x)$, one obtains the *D'Alembert solution*

$$u(t,x) = \frac{1}{2}[f(x+ct) + f(x-ct)] + \frac{1}{2c}\int_{x-ct}^{x+ct} g(s)\,\mathrm{d}s$$