MATH529

SP.22: Homework 2

Instructions

This homework assignment completes the review of ordinary differential equations, and introduces additional common Mathematica constructs. Exercises and problems are again from chapter 1 of Zill's textbook, and model solutions are presented.

In Mathematica, CTRL+SHIFT+9 opens an equation field in the internal Mathematica format, e.g., $\partial_x f(x)$. This allows an alternative to LaTeX equation input. Feel free to use whichever is easier. Export of equations in internal format to PDF is however sometimes dependent on system-installed fonts.

Exercises

From p.17: 18-28

Model solution: 1.2.17 (p. 17)

Statement

Determine $x\,y$ region in which $y' = y^{2/3}$ would have an unique solution passing through (x_0, y_0).

Solution

Equation $y' = f(y) = y^{2/3}$ must be continuous with continuous derivative. Since f is continuous everywhere, and f' is discontinuous only at $y = 0$, the DE has an unique solution throughout the real plane except for initial conditions of the form $(x_0, 0)$

In[ ]:=
```
DSolve[{y'[x] == y[x]^(2/3), y[0] == 1}, y[x], x]
```

Out[ ]=

$$\left\{\left\{y[x] \to \frac{1}{27} \times \left(27 + 27\,x + 9\,x^2 + x^3\right)\right\}\right\}$$

Problems

From p.31: 15-43

Model solution: 1.R.15 (p. 31)

Statement

Interpret the following statement as a DE:

On the graph of $y = \phi(x)$, the slope of the tangent at point $P(x, y)$ is the square of the distance from $P(x, y)$ to the origin

Solution

The slope is given by y', and the distance from $P(x, y)$ to the origin is $\sqrt{x^2 + y^2}$, giving DE

$$y' = x^2 + y^2$$

a first-order Bernoulli equation.

Model solution: 1.R.31 (p. 31)

Statement

Verify that $x^3\, y^3 = x^3 + 5$ is an implicit solution of $x\, y' + y = 1 / y^2$.

Solution

For $x \neq 0$, differentiate the solution to obtain

$3\,(y + x\,y')\,x^2\,y^2 = 3\,x^2$, which reduces to $x\,y' + y = 1 \big/ y^2$ for $y \neq 0$.

In[ ]:=
```
sol = x^3 (y[x])^3 == x^3 + 5
```

Out[ ]=
```
x^3 y[x]^3 == 5 + x^3
```

In[ ]:=
```
eq = Assuming[x ≠ 0, FullSimplify[D[sol, x]]]
```

Out[ ]=
```
y[x]^2 (y[x] + x y'[x]) == 1
```

In[ ]:=
```
Assuming[y[x] ≠ 0, Simplify[MultiplySides[eq, y[x]^-2]]]
```

Out[ ]=
```
y[x] + x y'[x] == 1/y[x]^2
```