MATH529: Linear algebra for applications

Homework 11

This assignment is a worksheet of exercises intended as preparation for the Final Examination. You should:

1. Review separation of variable techniques for PDEs

2. Set aside 60 minutes to solve these exercises. Each exercise is meant to be solved within 10 minutes. If you cannot find a solution within 3 minutes, skip to the next one.

3. Check your answers in Mathematica. Revisit theory for skipped or incorrectly answered exercies.

Per course policy, your best 10 homework scores enter into your final grade. Homeworks 11 and 12 are thus considered supplementary assignments. Nonetheless, everyone should attempt these as final examination preparation.

1. Find the eigenvalues and eigenfunctions of
$$x^2 \, y'' + x \, y' + \lambda y = 0, \, y'(1) = 0, \, y(e) = 0.$$

2. Solve the problem
$$u_{tt} = c^2 \, u_{xx}, \, u(0, t) = 0, \, u(L, t) = 0, \, u(x, 0) = 0, \, u_t(x, 0) = x(1 - x).$$

3. On rectangle $[0, a] \times [0, b] \subset \mathbb{R}^2$ solve the problem
$$u_{xx} + u_{yy}, \, u(0, y) = 0, \, u(a, y) = 0, \, u(x, 0) = f(x), \, u(x, b) = g(x).$$

4. Formulate the problem of finding the temperature distribution within a spherical ball of radius $R = 1$ at initial temperature $u(r, 0) = 0$ dropped into a liquid of temperature $U = 100$.

5. Solve the above problem.