

HOMEWORK 12

This assignment is a worksheet of exercises intended as preparation for the Final Examination. You should:

1. Review complex analysis techniques
2. Set aside 60 minutes to solve these exercises. Each exercise is meant to be solved within 10 minutes. If you cannot find a solution within 3 minutes, skip to the next one.
3. Check your answers in Mathematica. Revisit theory for skipped or incorrectly answered exercises.

Per course policy, your best 10 homework scores enter into your final grade. Homeworks 11 and 12 are thus considered supplementary assignments. Nonetheless, everyone should attempt these as final examination preparation.

1. For $z \in \mathbb{C}$ find the real and imaginary parts of $\tan z$.
2. At what points within \mathbb{C} is the function $f(z) = 2x^2 + y + i(y^2 - x)$ differentiable? At what points is it analytic?
3. Prove that

$$\text{ArcTan } z = \frac{i}{2} \ln \frac{i+z}{i-z}$$

4. What is the Laurent series of

$$f(z) = \frac{1 - e^{iz}}{z^4}, |z| > 0?$$

5. Determine the value of the integral

$$I = \int_0^{2\pi} \frac{\cos^2 \theta}{2 + \sin \theta} d\theta.$$