MATH529 Lesson02

First-order differential equations

Development of differential equation theory is guided by analysis of generic cases. The first generic case is that of first-order differential equations given in explicit form with an initial condition, known as the initial value problem (IVP)
$y' = dy / dx = f(x, y),\ \ y(x_0) = y_0$

A unique solution exists over some interval $R = (x_0, x_1) \times (y_0, y_1)$ if f is continuous over $\mathbb{R}$, and the derivative of f is continuous on $\mathbb{R}$. (a weaker sufficient condition is Lipschitz continuity in dependence of f on y)

An example of an IVP with multiple solutions
$y' = dy / dx = x\, y^{1/2},\ \ y(0) = 0, f(x, y) = x\, y^{1/2}$
$\partial f / \partial y = x \big/ \left(2\, y^{1/2}\right)$

```
DSolve[{y'[x] == x (y[x])^(1/2), y[0] == 0}, y[x], x]
```

Out[ ]= $\left\{\left\{y[x] \to \frac{x^4}{16}\right\}\right\}$

In[ ]:=
```
DE = y'[x] == x (y[x])^(1/2)
```

Out[ ]= $y'[x] == x \sqrt{y[x]}$

In[ ]:=
```
sol = DSolve[{DE, y[0] == 0}, y[x], x][[1, 1]]
```

Out[ ]= $y[x] \to \frac{x^4}{16}$

In[ ]:=
```
dsol = y'[x] → x^3 / 4
```

Out[ ]= $y'[x] \to \frac{x^3}{4}$

In[ ]:=
```
Assuming[x > 0, Simplify[DE /. {sol, dsol}]]
```

Out[ ]=
```
True
```

In[ ]:=
```
{a, b} /. {a → 1, b → 2}
```

Out[ ]=
```
{1, 2}
```

Separable first-order equations

Linear first-order equations