MATH529 Lesson02

First-order differential equations

Development of differential equation theory is guided by analysis of generic cases. The first generic case is that of first-order differential equations given in explicit form with an initial condition, known as the initial value problem (IVP) $y' = dy / dx = f(x, y), y(x_0) = y_0$

A unique solution exists over some interval $R = (x_0, x_1) \times (y_0, y_1)$ if f is continuous over \mathbb{R} , and the derivative of f is continuous on \mathbb{R} . (a weaker sufficient condition is Lipschitz continuity in dependence of f on y)

An example of an IVP with multiple solutions $y' = dy / dx = x y^{1/2}, y(0) = 0, f(x, y) = x y^{1/2}$ $\partial f / \partial y = x / (2 y^{1/2})$

DSolve
$$[\{ y'[x] = x (y[x])^{1/2}, y[0] = 0 \}, y[x], x]$$

Out[•]=

In[•

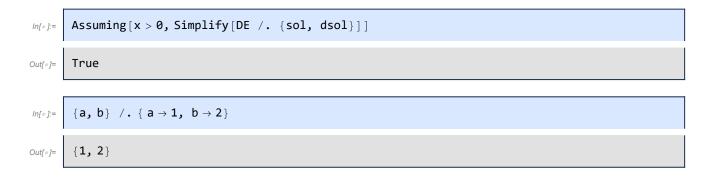
$$DE = y'[x] = x (y[x])^{1/2}$$

 $\left\{\left\{y\left[x\right]\rightarrow\frac{x^{4}}{16}\right\}\right\}$

 $Out[\bullet] = \mathbf{y}'[\mathbf{x}] = \mathbf{x} \sqrt{\mathbf{y}[\mathbf{x}]}$

$$Out[\bullet]= \left[\begin{array}{c} y[x] \rightarrow \frac{x^4}{16} \end{array} \right]$$

$$ln[*]:= \left[\begin{array}{c} dsol = y'[x] \rightarrow x^3/4 \\ \\ Out[*]= \end{array} \right] y'[x] \rightarrow \frac{x^3}{4} \end{array}$$



Separable first-order equations

Linear first-order equations