Overview

- Potential flow
- Flow around a circle
- Flow around Zhukovsky (Joukowsky) airfoil
- Honors, course capstone project

- Flow of an incompressible, inviscid fluid is described by the PDEs

$$\nabla \cdot \boldsymbol{v} = 0, \boldsymbol{v}_t + (\boldsymbol{v} \cdot \nabla)\boldsymbol{v} = -\nabla p$$

  expressing conservation of mass (continuity equation), momentum (Euler equation, a continuum formulation of Newton's law $\mathrm{d}(m\boldsymbol{v})/\mathrm{d}t = \sum \boldsymbol{f}$).

- The continuity equation implies $\exists \varphi$ such that $\boldsymbol{v} = \nabla\varphi$, and $\nabla^2\varphi = 0$, i.e., the hydrodynamic potential φ is harmonic

- Complex formulation:
  - Conformal map from domain $Z = X + iY$ to $z = x + iy$, e.g., $z = Z + 1/Z$
  - Complex potential $F(Z) = \Phi(X, Y) + i\Psi(X, Y)$, $f(z) = F(Z(z))$
  - Complex velocity $W = \mathrm{d}F/\mathrm{d}Z$, $w = \mathrm{d}f/\mathrm{d}z$

$$w(z) = \frac{\mathrm{d}F(Z(z))}{\mathrm{d}z} = \frac{\mathrm{d}F}{\mathrm{d}Z}\frac{\mathrm{d}Z}{\mathrm{d}z} = \frac{\frac{\mathrm{d}F}{\mathrm{d}Z}}{\frac{\mathrm{d}z}{\mathrm{d}Z}}$$

- Potential in Z plane around a circle $F(Z) = U_\infty(e^{i\alpha}Z + e^{-i\alpha}/Z)$
  - $F = \Phi + i\Psi$, Φ real velocity potential, Ψ real streamline function
  - $F(e^{i\theta}) = U_\infty(e^{i\alpha}e^{i\theta} + e^{-i\alpha}e^{-i\theta}) = 2U_\infty\cos(\alpha+\theta) \in \mathbb{R} \Rightarrow \Psi = 0 = \text{constant}$, the circle $Z = e^{i\theta}$ is a streamline
- Potential with circulation $F(Z) = U_\infty(e^{i\alpha}Z + e^{-i\alpha}/Z) + i\Gamma/Z$, the circulation Γ introduces asymmetry between velocity fields on circle top/bottom, a mathematical model of lift.

- Conformal maps can be found between simple domains (e.g., a circle or the upper half plane) and shapes of practical interest, such as airfoils
- The Joukowski transform maps a circle onto air and hydrofoil shapes

$$z = Z + \frac{1}{Z}$$

  - $Z = e^{i\Theta} \Rightarrow z = e^{i\Theta} + e^{-i\Theta} = 2\cos\Theta$, e.g., a flate plate
  - $Z = e^{i\Theta} + i\delta, \delta \in (0, 1/2)$ a cambered, thick airfoil, symmetric fore-aft
  - $Z = e^{i\Theta} + \rho e^{i\phi}, \rho \in (0.2, 0.4), \phi \in (-\pi/2, -\pi/4)$ cambered, thick, unsymmetrical airfoils, used in aircraft design 1920's

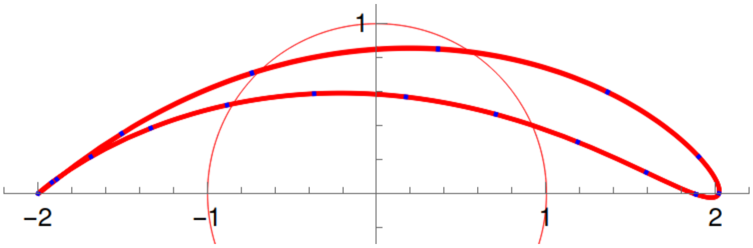

Figure 1. $\rho = 0.3, \phi = -1.38$ airfoil from Joukowski map