MATH529 Lesson01: ODE Review

Chapter 1 Introduction to ODE review

ODE:

Explicit form $y^{(n)} = f\left(x, y, y', \dots, y^{(n-1)}\right)$

Implicit form $0 = F\left(x, y, y', \dots, y^{(n-1)}, y^{(n)}\right)$

First order ODEs, n = 1, explicit form $y' = f(x, y)$

Separable first-order ODE $y' = g(x)\, h(y) = \frac{dy}{dx}$, $g(x)\, dx = \frac{dy}{h(y)}$

Separable ODEs

p.30, Ex. 7 $x\, y' = 2\, y$, $y' = 2\, \frac{y}{x}$, $g(x) = \frac{1}{x}$, $h(y) = 2\, y$, $\frac{1}{x}\, dx = \frac{dy}{2\, y}$

Solve by integration on both side $\int \frac{1}{x}\, dx = \int \frac{1}{2\, y}\, dy$, $\log(x) = \frac{1}{2} \log(y) - C$

In[ ]:=
```
g[x_] = 1/x; h[y_] = 2 y;
```

In[ ]:=
```
Integrate[g[x], x]
```

Out[ ]=
```
Log[x]
```

In[ ]:=
```
Integrate[1 / h[y], y]
```

Out[ ]=
$$\frac{\text{Log}[y]}{2}$$

In[ ]:=
```
DiffEq = D[y[x], x] == g[x] × h[y[x]]
```

Out[ ]=
$$y'[x] == \frac{2\,y[x]}{x}$$

In[ ]:=
```
DSolve[DiffEq, y[x], x]
```

Out[ ]=
$$\{\{y[x] \to x^2\, c_1\}\}$$

p.30, Ex 11, $y'' + 9y = 18$, $y'' = 18 - 9y$
Homogeneous linear ODE of order n, $y^{(n)} + a_{n-1}\,y^{(n-1)} + a_{n-2}\,y^{(n-2)} + \ldots + a_1\,y' + a_0\,y = 0$
Guess solution to be of form $y(x) = e^{rx}$, obtain $\left(r^n + a_{n-1}\,r^{n-1} + a_{n-2}\,r^{n-2} + \ldots + a_1\,r + a_0\right)e^{rx} = 0$, implying that $p_n(r) = r^n + a_{n-1}\,r^{n-1} + a_{n-2}\,r^{n-2} + \ldots + a_1\,r + a_0 = 0$

Step 1: Solve the homogeneous problem $y'' + 9y = 0$, characteristic equation, $r^2 + 9 = 0$, with roots $r_1 = 3i$, $r_2 = -3i$. Solution is $y_h(x) = c_1\,e^{3ix} + c_2\,e^{-3ix} = A\cos(3x) + B\sin(3x)$

Step 2: State that the solution is the sum of the homogeneous solution and a particular solution $y = y_h + y_p$

In[ ]:=
```
DiffEqH = D[y[x], {x, 2}] + 9 y[x] == 0
```
Out[ ]=
```
9 y[x] + y''[x] == 0
```
In[ ]:=
```
DSolve[DiffEqH, y[x], x]
```
Out[ ]=
```
{{y[x] → c1 Cos[3 x] + c2 Sin[3 x]}}
```
In[ ]:=
```
DiffEq = D[y[x], {x, 2}] + 9 y[x] == 18
```
Out[ ]=
```
9 y[x] + y''[x] == 18
```
In[ ]:=
```
DSolve[DiffEq, y[x], x]
```
Out[ ]=
```
{{y[x] → 2 + c1 Cos[3 x] + c2 Sin[3 x]}}
```

p.30, Ex. 12. $x\,y'' - y' = 0$, Let $y' = z$, $x\,z' = z$, a separable ODE $x\,\frac{dz}{dx} = z$, $x\,\frac{dz}{z} = \frac{dx}{x}$ with solution $z(x) = C\,x$, and $y(x) = \frac{1}{2}\,C\,x^2 + D$

In[ ]:=
```
DiffEq = x y''[x] - y'[x] == 0
```
Out[ ]=
```
-y'[x] + x y''[x] == 0
```
In[ ]:=
```
DSolve[DiffEq, y[x], x]
```
Out[ ]=

$$\left\{\left\{y[x] \to \frac{x^2\,c_1}{2} + c_2\right\}\right\}$$

p.31, Ex. 34, $(1 + x\,y)\,y' + y^2 = 0$. Consider $y = e^{-x\,y}$

In[ ]:=
```
Subst = y[x] == Exp[-x y[x]]
```
Out[ ]= $y[x] == e^{-x\, y[x]}$

In[ ]:=
```
result = Simplify[D[Subst, x]]
```
Out[ ]= $y'[x] == -e^{-x\, y[x]} (y[x] + x\, y'[x])$

In[ ]:=
```
verif = result /. e^(-x y[x]) → y[x]
```
Out[ ]= $y'[x] == -y[x] (y[x] + x\, y'[x])$

In[ ]:=
```
DiffEq = (1 + x y[x]) y'[x] + (y[x])^2 == 0
```
Out[ ]= $y[x]^2 + (1 + x\, y[x])\, y'[x] == 0$

In[ ]:=
```
DSolve[DiffEq, y[x], x]
```
Solve : Inverse functions are being used by Solve , so some solutions may not be found ; use Reduce for complete solution information .

Out[ ]= $\left\{\left\{y[x] \to \frac{\text{ProductLog}[e^{c_1} x]}{x}\right\}\right\}$

In[ ]:=
```
? ProductLog
```
Out[ ]=

Symbol

ProductLog [z] gives the principal solution for w in $z = w e^{w}$.

ProductLog [k, z] gives the k^{th} solution .

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Attributes {Listable , Protected , ReadProtected }

Full Name System`ProductLog