# MATH547 Final Examination

Fall 2014 Semester, December 9, 2014

**Instructions**. *Answer the following questions. Provide a motivation of your approach and the reasoning underlying successive steps in your solution. Write neatly and avoid erasures. Use scratch paper to sketch out your answer for yourself, and then transcribe your solution to the examination you turn in for grading. Illegible answers are not awarded any credit. Presentation of calculations without mention of the motivation and reasoning are not awarded any credit. The last, seventh question is optional and offered to enable raising your score on the midterm examination. Each complete, correct solution to an examination question is awarded 4 course grade points. Your primary goal should be to demonstrate understanding of course topics and skill in precise mathematical formulation and solution procedures.*

1. Consider a block partitioning of a square matrix $M \in \mathbb{R}^{m \times m}$

$$M = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \tag{1}$$

with $A, B, C, D$ compatible submatrices. Compute

$$\begin{pmatrix} I & 0 \\ -CA^{-1} & I \end{pmatrix} M. \tag{2}$$

2. Again consider the block partitioning (1) of matrix $M \in \mathbb{R}^{m \times m}$. Use the result from (2) and the identity

$$\begin{vmatrix} A & B \\ 0 & D \end{vmatrix} = |A|\,|D| \tag{3}$$

to prove that if $AC = CA$ then

$$\det(M) = |M| = |AD - CB|.$$

3. Consider the matrix $V \in \mathbb{R}^{3 \times 3}$

$$V = \begin{pmatrix} 1 & 1 & 3 \\ 1 & -2 & 0 \\ 1 & 1 & -3 \end{pmatrix}$$

with mutually orthogonal column vectors $V_1, V_2, V_3 \in \mathbb{R}^3$. What is the volume of the parallelepiped with edges $V_1, V_2, V_3$?

4. Find the eigenvalues and unit eigenvectors of

$$A = \begin{pmatrix} 2 & 2 & 2 \\ 2 & 0 & 0 \\ 2 & 0 & 0 \end{pmatrix}.$$

5. Show that the eigenvalues of a skew-symmetric real matrix, $A \in \mathbb{R}^{m\times m}$, $A^T = -A$ are purely imaginary.

6. Compute the singular value decomposition of

$$A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{pmatrix}.$$

What is rank$(A)$?

7. Revisit the midterm problem of computing $A^{200}$ with

$$A = \begin{pmatrix} 1 & 0 & 1/2 & 1/2 \\ 0 & 1 & 1/2 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \\ 0 & 0 & 1/2 & 1/2 \end{pmatrix}.$$

Use the eigendecomposition of $A = X\Lambda X^{-1}$ to compute $A^{200}$.