

MATH547 Midterm Examination

Fall 2015 Semester, October 9, 2015

Instructions. Answer the following questions. Provide a motivation of your approach and the reasoning underlying successive steps in your solution. Write neatly and avoid erasures. Use scratch paper to sketch out your answer for yourself, and then transcribe your solution to the examination you turn in for grading. Illegible answers are not awarded any credit. Presentation of calculations without mention of the motivation and reasoning are not awarded any credit. Each complete, correct solution to an examination question is awarded 4 course grade points. Your primary goal should be to demonstrate understanding of course topics and skill in precise mathematical formulation and solution procedures.

1. Show that the vectors

$$\mathbf{u}_1 = \begin{pmatrix} 1 \\ 0 \\ 2 \\ 1 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} -2 \\ 3 \\ -1 \\ 1 \end{pmatrix}, \mathbf{u}_3 = \begin{pmatrix} 2 \\ -2 \\ 1 \\ -1 \end{pmatrix},$$

are linearly independent.

2. Construct an orthonormal basis for the span of the vectors

$$\mathbf{u}_1 = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \mathbf{u}_2 = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \mathbf{u}_3 = \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}.$$

3. Find the straight line $y = ax + b$ that best fits the following data in the least squares sense:

i	1	2	3	4
x_i	0	1	3	6
y_i	2	3	7	12