Homework 0

Due date: January 20, 2017, 11:55PM.

Bibliography: Course lecture notes Lessons 1-4.

Homework 0 is a test of submission procedures, and is not included in course grade. All homework assignments will follow the same format. Whereas normal course points typically require 10-15 mins of work (approximately 2 hours per assignment) once the relevant theory is understood, the exercises in this homework are very simple and only require a minute or two. Problems 1,3 show the solution. Problems 2,4 ask you to do a similar task. The solutions show the expected formatting to be used in future homework submissions. Comments on the formatting and expectations are shown in red. Problem 5 is a computational exercise, again showing part of the solution.

1. Find the coordinates of $\boldsymbol{b}=(\ 7\ \ 10\ )^T$ as a linear combination of the vectors $\boldsymbol{a}_1=(\ 2\ \ -1\ )^T$, $\boldsymbol{a}_2=(\ 3\ \ 3\ )^T$.

   Solution. Express the linear combination as $\boldsymbol{b}=x_1\boldsymbol{a}_1+x_2\boldsymbol{a}_2$, or as the matrix-vector multiplication

$$\boldsymbol{A}\boldsymbol{x}=\boldsymbol{I}\boldsymbol{b}=\boldsymbol{b}, \boldsymbol{A}\in\mathbb{R}^{2\times 2}, \boldsymbol{b},\boldsymbol{x}\in\mathbb{R}^2. \quad (1)$$

First step of your solution is a concise statement of the relevant theoretical concept, in this case, definition of a linear combination, and its expression as a matrix-vector product. Always show the dimension of vectors and matrices as above. In the above text some of the formatting is carried out in text mode, and some in math mode. You enter math mode through the dollar sign character. Refer to scicomp.web.unc.edu turorial videos on use of TeXmacs. Note the consistent use of notation:

- Upper-case, Latin bold letters for matrices $\boldsymbol{A},\boldsymbol{B},\ldots$
- Lower-case, Latin bold letters for vectors: $\boldsymbol{a}_1,\boldsymbol{b},\boldsymbol{x}$
- Bold, lower-case Latin letters with a single index for the column vector of the matrix denoted by the corresponding upper-case Latin bold symbol: $\boldsymbol{A}=(\ \boldsymbol{a}_1\ \ \boldsymbol{a}_2\ )$

Mathematical statements are sentences, and you should use punctuation just as in English sentences. The above statement is read: "Matrix $\boldsymbol{A}$ times vector $\boldsymbol{b}$ equals matrix $\boldsymbol{I}$ times vector $\boldsymbol{b}$, and also equals vector $\boldsymbol{b}$ with matrix $\boldsymbol{A}$ 2 by 2 with real components, vectors $\boldsymbol{b},\boldsymbol{x}$ with 2 real components.".

The vector equality (1) corresponds to the system

$$\left\{\begin{array}{rcl} 2x_1+3x_2 & = & 7 \\ -x_1+3x_2 & = & 10 \end{array}\right.,$$

with solution $x_1=-1$, $x_2=3$.

2. Find the coordinates of $\boldsymbol{b}=(\ 0\ \ 10\ )^T$ as a linear combination of the vectors $\boldsymbol{a}_1=(\ 3\ \ -1\ )^T$, $\boldsymbol{a}_2=(\ 1\ \ 3\ )^T$.

3. Prove $(\boldsymbol{a}+\boldsymbol{b})^T=\boldsymbol{a}^T+\boldsymbol{b}^T$, $\boldsymbol{a},\boldsymbol{b}\in\mathbb{R}^m$.

   Solution. By definition of vector addition

$$\boldsymbol{a}+\boldsymbol{b}=\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_m \end{pmatrix}+\begin{pmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{pmatrix}=\begin{pmatrix} a_1+b_1 \\ a_2+b_2 \\ \vdots \\ a_m+b_m \end{pmatrix}, \quad (2)$$

and, also using definition of transpose,

$$\boldsymbol{a}^T+\boldsymbol{b}^T=(\ a_1\ \ a_2\ \ \ldots\ \ a_m\ )+(\ b_1\ \ b_2\ \ \ldots\ \ b_m\ )=(\ a_1+b_1\ \ a_2+b_2\ \ \ldots\ \ a_m+b_m\ ). \quad (3)$$

First, state the relevant theory. Note that we use the column vector convention: "vector" without any other qualifier means column vector.

Again by definition of transpose

$$(\boldsymbol{a}+\boldsymbol{b})^T = \begin{pmatrix} a_1+b_1 & a_2+b_2 & \ldots & a_m+b_m \end{pmatrix}, \tag{4}$$

the same quantity as in rhs of eq (3), q.e.d.

Use link/references (from Insert->Link menu) to refer to specific equations, figures. Use standard acronyms, e.g., rhs=right hand side, q.e.d.="quod erat demonstrandum"="what was to be proven".

4. Prove $(\boldsymbol{A}\boldsymbol{x})^T = \boldsymbol{x}^T\boldsymbol{A}^T$, $\boldsymbol{x}\in\mathbb{R}^m$, $\boldsymbol{A}\in\mathbb{R}^{m\times n}$.

5. (Computer application). This example computer application is also meant as an introduction to use of Octave (Matlab) in linear algebra. Tutorial comments on use of Octave are colored in green).

Use Octave to compute the linear combination

$$\boldsymbol{r} = a\boldsymbol{u}+b\boldsymbol{v}+c\boldsymbol{w}, \text{with}\, a=4, b=-2, c=1, \boldsymbol{u} = \begin{pmatrix} 2\\ -3\\ 4\\ 1\\ 0 \end{pmatrix}, \boldsymbol{v} = \begin{pmatrix} 1\\ 2\\ -5\\ 2\\ 4 \end{pmatrix}, \boldsymbol{w} = \begin{pmatrix} -1\\ 3\\ 0\\ 1\\ 2 \end{pmatrix}.$$

Octave uses square brackets for vectors, whereas in TeXmacs we use paranthesis. Commands in Octave are terminated by a semicolon. In Octave a vector is defined by square brackets, and displayed using the disp function. Components along a row are separated by spaces. Examples:

```
octave> x=[1 2 3]; disp(x);
    1   2   3
octave>
```

and rows are separated by semicolons inside the vector brackets.

```
octave> y=[1;2;3]; disp(y);
    1
    2
    3
octave>
```

The transposition operation is defined by an apostrophe

```
octave> disp(y');
    1   2   3
octave>
```

Define the three vectors $\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$

```
octave> u=[2 -3 4 1 0]'; v=[1 2 -5 2 4]'; w=[-1 3 0 1 2]';
octave>
```

Define the three scalars

```
octave> a=4; b=-2; c=1;
```

Compute the result and display it

```
octave> r=a*u+b*v+c*w; disp(r);
     5
   -13
    26
     1
    -6
octave>
```

Repeat the above in Octave, for scalars $x = -1$, $y = 3$, $z = 0$.