HOMEWORK 7&8 COMBINED

Due date: April 28, 2017, 11:55PM.

Bibliography: Course lecture notes Lessons 23-26.

Note: Save this file as `Homework7&8Solution.tm` in your `/home/student/courses/MATH547/homework` directory. Then carry out your work on solving the problems in TeXmacs. Submit your final version through Sakai. If you've understood the theoretical concepts reviewed in the Background section and lecture notes, each problem should require about 10 minutes to complete, hence the entire homework can be completed within 3 hours.

1 Background

This assignment serves as a course review and final examination preparation. You may choose to solve either 16 theory problems or 8 theory problems and a capstone computational application on model reduction using the SVD. Each theory exercise is indicative of what you can expect to find on the final examination, and should take no more than 10 minutes to complete if the underlying theory is well understood. Though the questions themselves are drawn from material from the second half of the semester (Lessons 20-28), mastery of concepts from the first half of the semester (e.g., matrix operations, matrix fundamental spaces) is required for insightful solutions. The computational application can also be readily completed in 90 minutes if the theory is well understood.

2 Theory problems

1. *Ex7.1.5 (p.323).*
2. *Ex7.1.6 (p.323).*
3. *Ex7.1.19 (p.324).*
4. *Ex7.1.21 (p.324).*
5. *Ex7.1.45 (p.325).*
6. *Ex7.1.47 (p.325).*
7. *Ex7.2.14 (p.336).*
8. *Ex7.2.16 (p.336).*
9. *Ex7.2.18 (p.336).*
10. *Ex7.2.23 (p.336).*
11. *Ex7.3.21 (p.345).*
12. *Ex7.3.25 (p.345).*
13. *Ex7.3.28 (p.345).*
14. *Ex8.3.16 (p.412).*
15. *Ex8.3.19 (p.412).*
16. *Ex8.3.20 (p.412).*

3 Computational problems

Hyperbranched polymers (Fig. 1) arise in many biological settings. The mechanical properties of the polymers very often determine their biological role in an organism

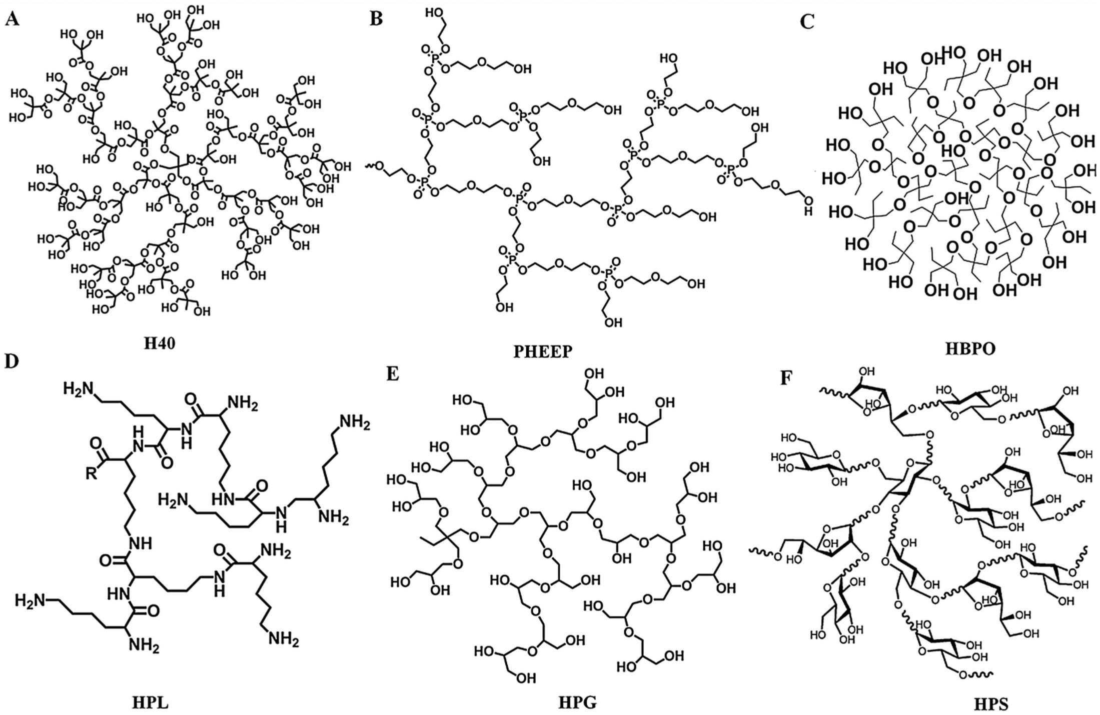

Figure 1. Typical hyperbranched polymers

1. Choose one of the polymers from Fig. 1. Consider it to be a two-dimensional structure. Print out the chosen polymer, superimpose transparent graph paper and construct a vector of coordinates for the polymer $\boldsymbol{x} \in \mathbb{R}^{2N}$, where N is the number of nodes in the structure. You may also visually estimate the coordinates. This will be the most time-consuming part of the project.
2. Assign masses (based on node molecular weights) and stiffnesses to springs connecting each node. Construct the system

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{0}$$

   describing free vibrations of the polymer in Octave
3. Consider vibrations of the polymer described as $\boldsymbol{x}(t) = \sin(\omega t)\,\boldsymbol{y}$. Solve the eigenproblem

$$(\boldsymbol{M}^{-1}\boldsymbol{K})\,\boldsymbol{y} = \boldsymbol{A}\boldsymbol{y} = \omega^2\,\boldsymbol{y}$$

   and plot 10 of the eigenmodes $\boldsymbol{u}$ (each eigenmode contains coordinates of the motion of the node during vibrational motion).
4. Construct the SVD $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T$. Choose the dominant k singular vectors to obtain $\tilde{\boldsymbol{A}} = \tilde{\boldsymbol{U}}\tilde{\boldsymbol{\Sigma}}\tilde{\boldsymbol{V}}^T$ ($\tilde{\boldsymbol{U}}$ contains only k columns, similar for other matrices). Find the eigenmodes of

$$\tilde{\boldsymbol{A}}\tilde{\boldsymbol{y}} = \omega^2\tilde{\boldsymbol{y}},$$

   and plot them superimposed on those from question 3. Comment on the result.