Homework 4

Due date: Mar 10, 2017, 11:55PM.

Bibliography: Course lecture notes Lessons 11-15. Textbook sections 6.1-6.3

Note: Save this file as `Homework4Solution.tm` in your `/home/student/courses/MATH547/homework` directory. Then carry out your work on solving the problems in TeXmacs. Submit your final version through Sakai. If you've understood the theoretical concepts reviewed in the Background section and lecture notes, each problem should require about 20 minutes to complete, hence the entire homework can be completed within 3 hours.

1 Background

1.1 Objectives, basic concepts

This homework explores the links between computational geometry and linear algebra. You perhaps have been exposed to solving linear systems by using determinants and Cramer's rule. There are multiple reasons why this approach, suitable for small systems and solution by hand, is not recommended when considering realistic linear algebra applications. Determinants do find extensive use in computational geometry, as will be explored in this homework.

2 Theory problems

2.1 Determinant calculation rules

Solve exercises 6.1.17-22, from textbook, p.275. Solution to exercise 6.1.19 is presented as an example. Present solutions of the other exercises. You should do these both by hand and check using computer algebra. Basic operations within the *Mathematic*a computer algebra system are introduced.

Ex 6.1.19 Solution. The determinant of matrix

$$\boldsymbol{A} = \begin{pmatrix} 1 & 1 & k \\ 1 & k & k \\ k & k & k \end{pmatrix}$$

can be computed in various ways. The most efficient procedure is to linearly combine columns and/or rows to form zeros along a row/column and then expand the determinant.

$$\begin{vmatrix} 1 & 1 & k \\ 1 & k & k \\ k & k & k \end{vmatrix} \overset{C2 \leftarrow C2 - C1}{=} \begin{vmatrix} 1 & 0 & k \\ 1 & k-1 & k \\ k & 0 & k \end{vmatrix} \overset{\text{Expand}\,(2,2)}{=} (k-1) \begin{vmatrix} 1 & k \\ k & k \end{vmatrix} \overset{C1 \leftarrow C1 - C2}{=} (k-1) \begin{vmatrix} 1-k & k \\ 0 & k \end{vmatrix}.$$

Having reduced the determinant to that of an upper triangular matrix, obtain

$$\det \boldsymbol{A} = -k(k-1)^2, \det \boldsymbol{A} \neq 0 \Rightarrow k \neq 1 \text{ and } k \neq 0.$$

Matrices in *Mathematica* are represented as embedded lists enclosed in curly braces

```
Mathematica
In[1]:= A={{1,1,k},{1,k,k},{k,k,k}}; MatrixForm[A]
```

$$\begin{pmatrix} 1 & 1 & k \\ 1 & k & k \\ k & k & k \end{pmatrix}$$

```
In[3]:= f[k_]=Det[A]
```

$k\,(-k^2+2\,k-1)$

```
In[4]:= Solve[f[k]==0,k]
```

$\{\{k\to 0\},\{k\to 1\},\{k\to 1\}\}$

```
In[5]:=
```

Ex6.1.17-22 Solutions.

2.2 Cross products

Solve exercises 6.1.49, 6.1.50, p.276

Ex6.1.49 Solution.

Ex6.1.50 Solution.

2.3 Determinant properties

Solve 6.2.35, 6.2.40, 6.2.41, 6.2.42, p. 290-291

Ex6.2.35 Solution.

Ex6.2.40 Solution.

Ex6.2.41 Solution.

Ex6.2.42 Solution.

2.4 Geometric applications of determinants

Solve exercises 6.3.4, 6.3.5, 6.3.6, p.305-306.

Ex6.3.4 Solution.

Ex6.3.5 Solution.

Ex6.3.6 Solution.

3 Computational problems

Computational geometry extensively uses determinants. Complicated objects are built from unions of simpler objects: triangles and tetrahedra. Some example geometry files are available in the `MATH547/lessons/ply` directory. Here's a geometric description of an ant

```
octave> cd /home/student/courses/MATH547/lessons/ply
octave> [x,tri] = ply_to_tri_mesh('ant.ply');
octave> trisurf(tri', x(1,:), x(2,:), x(3,:) ); print -deps ant.eps;
```

```
octave>
```

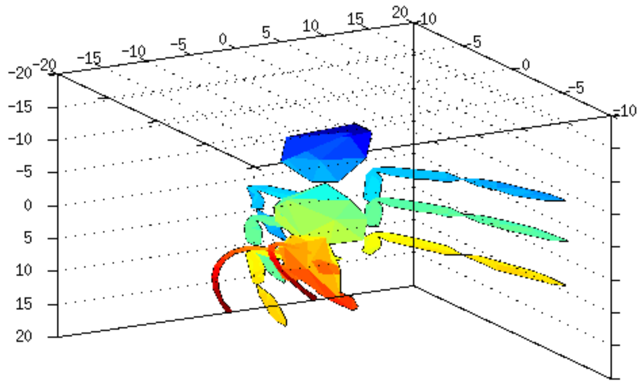

Figure 1. Ant from ply file

3.1 Surface areas

Choose one of the objects in the ply directory. Follow the procedure from Lesson18 and compute the surface area.

Solution. Continuing with the ant example, the computation follows the procedure from Lesson 18

```
octave> Area=0.;
octave> for n=1:max(size(tri))
          n1=tri(1,n); n2=tri(2,n); n3=tri(3,n);
          a1 = x(:,n2)-x(:,n1); a2 = x(:,n3)-x(:,n1);
          q1 = a1/norm(a1); h = a2 - (q1'*a2)*q1;
          base = norm(a1); height = norm(h);
          Area = Area + 0.5*base*height;
        end;
octave> Area
   Area =  904.90
octave>
```

3.2 Volumes

Compute the volume of the object.

Solution. Construct the tetrahedrals inside the ant and then sum volumes of the tetrahedra.

```
octave> tet=delaunay(x(1,:),x(2,:),x(3,:)); Volume=0.;
octave> for n=1:max(size(tet))
          n1=tet(n,1); n2=tet(n,2); n3=tet(n,3); n4=tet(n,4);
          A = [1 x(:,n1)'; 1 x(:,n2)'; 1 x(:,n3)'; 1 x(:,n4)']';
          Volume = Volume + abs(det(A))/6.;
        end;
octave> Volume
   Volume =  7394.1
octave>
```