

HOMEWORK 1

Due date: February 5, 2015, 11:55PM.

Bibliography: Course lecture notes Lessons 4-8. Textbook pp. 77-121, Sections 2.1-2.5.

1. (1 course point) Textbook p.87, Exercise 2.2.2.

a) $\mathcal{S} = \{(x, y, z)^T \mid x + y + z + 1 = 0\}$. Determine whether $\mathcal{S} \leq \mathbb{R}^3$. Choose two vectors $\mathbf{u}, \mathbf{v} \in \mathcal{S}$

$$\mathbf{u} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}, \mathbf{v} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \text{ with } x + y + z + 1 = 0, \text{ and } a + b + c + 1 = 0.$$

Compute

$$\mathbf{u} + \mathbf{v} = \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} a \\ b \\ c \end{pmatrix} = \begin{pmatrix} x + a \\ y + b \\ z + c \end{pmatrix}.$$

Verify $\mathbf{u} + \mathbf{v} \in \mathcal{S}$, compute $x + a + y + b + z + c + 1 = (x + y + z + 1) + (a + b + c + 1) - 1 = 0 + 0 - 1 = -1$, hence $\mathbf{u} + \mathbf{v} \notin \mathcal{S}$.

2. (1 course point) Textbook p.93, Exercises 2.3.4 and 2.3.7.

2.3.4.e. Verify $\text{span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\} = \mathbb{R}^2$ by double inclusion of sets. (1) $\forall \mathbf{v} \in \text{span}\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}, \exists x_1, x_2, x_3 \in \mathbb{R}$ such that $\mathbf{v} = x_1 \mathbf{a}_1 + x_2 \mathbf{a}_2 + x_3 \mathbf{a}_3 = \mathbf{A} \mathbf{x}$. Since $\mathbf{x} \in \mathbb{R}^3, \mathbf{A} = (\mathbf{a}_1 \mathbf{a}_2 \mathbf{a}_3) \in \mathbb{R}^{2 \times 3}, \mathbf{v} \in \mathbb{R}^2$

3. (1 course point) Textbook p.94, Exercises 2.3.8 and 2.3.9.

4. (1 course point) Textbook p.105, Exercises 2.4.2, 2.4.3, and 2.4.4.

(2.4.2.b) In Octave

```
octave> A=[0 -1 1; 1 3 3; -5 0 0];
octave> R=rref(A); disp(R);
    1    -0     0
    0     1     0
    0     0     1
octave> A=[0 -1 1; 4 0 -8; -1 1 1]; R=rref(A); disp(R);
    1     0    -2
    0     1    -1
    0     0     0
octave> disp(rank(A));
    3
octave>
```

5. (Computer application 4 course points) Textbook pp.105-106. Solve Exercises, 2.4.1, 2.4.2, 2.4.9 computationally by use of Octave.

2.4.9.c.

```
octave> A=[1 0 2 1; 0 1 -1 -2; -1 1 -3 1; 2 3 1 1]; disp(A);
    1     0     2     1
    0     1    -1    -2
   -1     1    -3     1
    2     3     1     1
octave> disp(rank(A));
    3
```

```

octave> disp(rank(A(:,2:4)));
3
octave> B=orth(A); disp(B);
0.519133 -0.150873 0.235007
-0.224857 0.257527 -0.833025
-0.528757 0.615369 0.495188
0.632737 0.729546 -0.075035
octave> B(:,2)'*B(:,1)
-1.1102e-16
octave> B(:,1)'*B(:,1)
1
octave> disp(B'*B)
1.0000e+00 -1.1102e-16 -2.7756e-17
-1.1102e-16 1.0000e+00 -1.1102e-16
-2.7756e-17 -1.1102e-16 1.0000e+00
texmacs
octave>

```