Homework 6

Due date: May 2, 2016, 10:00 AM.

This final homework is meant as a course review and final examination preparation. All questions are representative of what you can expect on the final exam.

1. (1 course point) Show that if $\boldsymbol{AC} = \boldsymbol{CA}$, then

$$\det\begin{pmatrix} \boldsymbol{A} & \boldsymbol{B} \\ \boldsymbol{C} & \boldsymbol{D} \end{pmatrix} = \det(\boldsymbol{AD} - \boldsymbol{CB}).$$

2. (1 course point) Compute the value of the determinant

$$\Delta = \begin{vmatrix} 9 & 1 & 9 & 9 & 9 \\ 9 & 0 & 9 & 9 & 2 \\ 4 & 0 & 0 & 5 & 0 \\ 9 & 0 & 3 & 9 & 0 \\ 6 & 0 & 0 & 7 & 0 \end{vmatrix}.$$

   Explain calculation steps.

3. (1 course point) Suppose $\boldsymbol{x}$ is an eigenvector of $\boldsymbol{A}$, corresponding to eigenvalue λ.
   a) Show that $\boldsymbol{x}$ is an eigenvector of $5\boldsymbol{I} - \boldsymbol{A}$. What is the corresponding eigenvalue?
   b) Show that $\boldsymbol{x}$ is an eigenvector of $5\boldsymbol{I} - 3\boldsymbol{A} + \boldsymbol{A}^2$. What is the corresponding eigenvalue?

4. (1 course point) Let

$$\boldsymbol{M} = \begin{pmatrix} \boldsymbol{A} & \boldsymbol{X} \\ \boldsymbol{0} & \boldsymbol{B} \end{pmatrix}.$$

   Use result from Problem 1 to prove that

$$p_{\boldsymbol{M}}(\lambda) = p_{\boldsymbol{A}}(\lambda) p_{\boldsymbol{B}}(\lambda),$$

   with $p_{\boldsymbol{M}}(\lambda), p_{\boldsymbol{A}}(\lambda), p_{\boldsymbol{B}}(\lambda)$, the characteristic polynomials of matrices $\boldsymbol{M}, \boldsymbol{A}, \boldsymbol{B}$.

5. (1 course point) Compute $\boldsymbol{A}^{100}$ for

$$\boldsymbol{A} = \begin{pmatrix} 3 & -1 \\ -1 & 3 \end{pmatrix}.$$

6. (1 course point) Compute the singular value decomposition of

$$\boldsymbol{A} = \begin{pmatrix} 3 & -3 \\ 0 & 0 \\ 1 & 1 \end{pmatrix}.$$

7. (1 course point) Compute the singular value decomposition of

$$\boldsymbol{A} = \begin{pmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{pmatrix}.$$

8. (1 course point) Consider $\boldsymbol{A} \in \mathbb{R}^{m \times m}$, symmetric.
   a) Show that $C(\boldsymbol{A})^\perp = N(\boldsymbol{A})$. ($\mathcal{S}^\perp$ is the orthogonal complement of vector space $\mathcal{S}$)
   b) Show that any $\boldsymbol{y} \in \mathbb{R}^m$ can be written as $\boldsymbol{y} = \boldsymbol{u} + \boldsymbol{v}$, with $\boldsymbol{u} \in C(\boldsymbol{A})$, and $\boldsymbol{v} \in N(\boldsymbol{A})$.