- Applications of determinants: volumes of simplicia

1. $d = 1$, define an interval by $\{\boldsymbol{r}_1, \boldsymbol{r}_2\}$, with $\boldsymbol{r}_1, \boldsymbol{r}_2 \in \mathbb{R}$, $\boldsymbol{r}_1 = (x_1)$, $\boldsymbol{r}_2 = (x_2)$. The signed length of the interval is $l = x_2 - x_1$ and can be computed as

$$\Delta = \begin{vmatrix} 1 & 1 \\ x_1 & x_2 \end{vmatrix}$$

2. $d = 2$, define a triangle by $\{\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3\}$, with $\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3 \in \mathbb{R}^2$. The signed area is

$$\Delta = \frac{1}{2} \begin{vmatrix} 1 & 1 & 1 \\ x_1 & x_2 & x_3 \\ y_1 & y_2 & y_3 \end{vmatrix}$$

3. $d = 3$, define a tetrahedron by $\{\boldsymbol{r}_1, \boldsymbol{r}_2, \boldsymbol{r}_3, \boldsymbol{r}_4\}$. The signed volume is

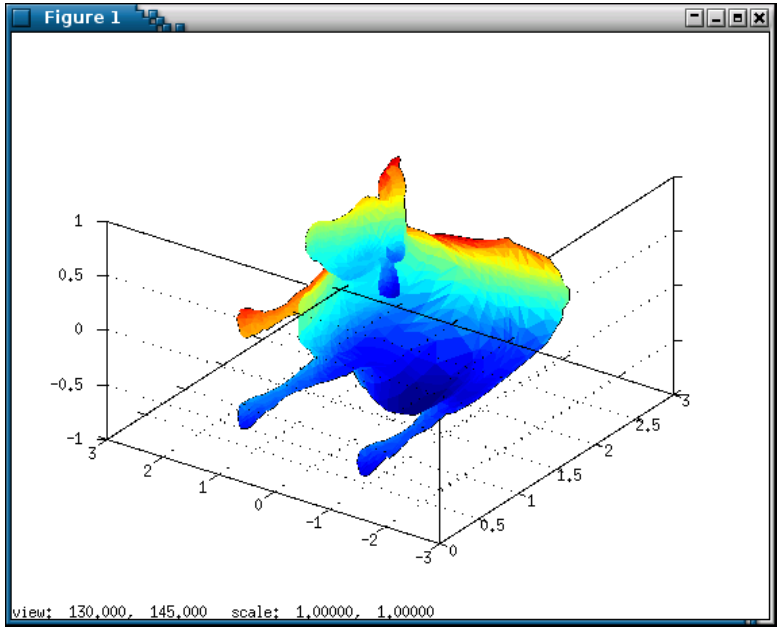
$$\Delta = \frac{1}{6} \begin{vmatrix} 1 & 1 & 1 & 1 \\ x_1 & x_2 & x_3 & x_4 \\ y_1 & y_2 & y_3 & y_4 \\ z_1 & z_2 & z_3 & z_4 \end{vmatrix}$$

Example: How big is the hide on that cow?

```
octave> cd ~/courses/MATH547/lessons/ply;
octave> [x,tri]=ply_to_tri_mesh('cow.ply');
octave> trisurf(tri', x(1,:), x(2,:), x(3,:) ); print -dpng cow.png;
octave> tri(:,100)'
ans =

   88   87  101

octave>
```



How big is the hide on that cow?

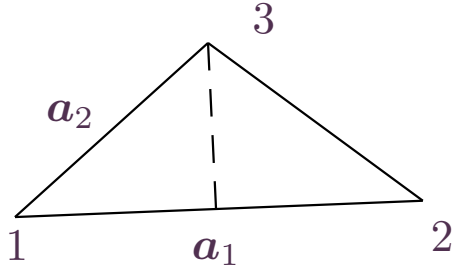
```
octave> Area=0.;
```

```
octave> for n=1:max(size(tri))
          n1=tri(1,n); n2=tri(2,n); n3=tri(3,n);
          a1 = x(:,n2)-x(:,n1); a2 = x(:,n3)-x(:,n1);
          q1 = a1/norm(a1); h = a2 - (q1'*a2)*q1;
          base = norm(a1); height = norm(h);
          Area = Area + 0.5*base*height;
        end;
```

```
octave> Area
```

Ahide = 21.193

```
octave>
```



$$\text{Area} = \frac{1}{2}\,(\text{base}) \times (\text{height})$$

$$A = \frac{1}{2}\,\|a_1\|\,\|h\| \qquad q_1 = \frac{1}{\|a_1\|}\,a_1$$

$$h = a_2 - (q_1^T a_2) q_1$$

How big is that cow?

```
octave> tet=delaunay(x(1,:),x(2,:),x(3,:));
```

```
octave> size(tet)
```

ans =

```
  19155      4
```

```
octave> Volume=0.;
```

```
octave> for n=1:max(size(tet))
          n1=tet(n,1); n2=tet(n,2); n3=tet(n,3); n4=tet(n,4);
          A = [1 x(:,n1)'; 1 x(:,n2)'; 1 x(:,n3)'; 1 x(:,n4)'];
          Volume = Volume + abs(det(A))/6.;
        end;
```

```
octave> Volume
```

Volume = 10.647

```
octave>
```