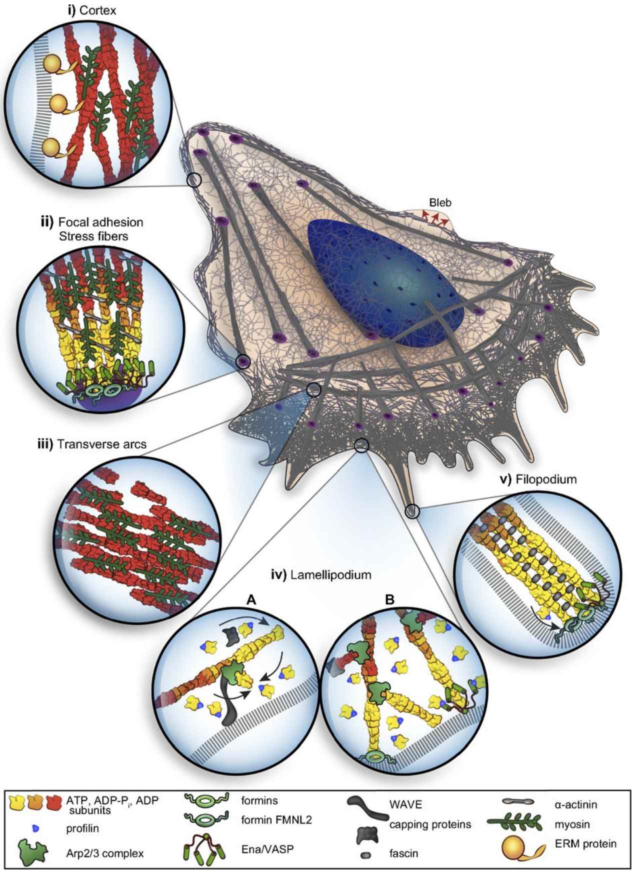
The following 3 lessons present realistic applications of linear algebra

1. Extracting information about cellular processes during movement using SVD
2. Web search algorithms (Page & Brin PageRank) using eigendecomposition
3. The search for exoplanets using least squares

- Cellular structure

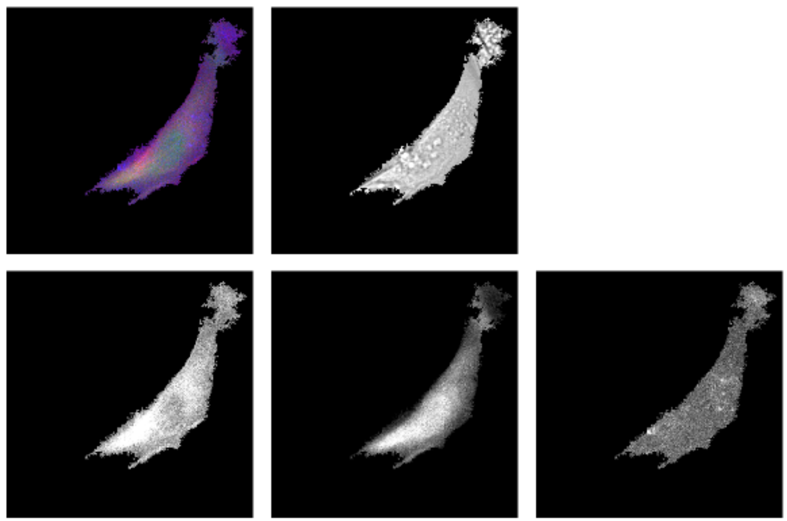


Cells move by reconfiguration of an internal structure of microtubules, actin filaments under the action of molecular motors

Experimental data on cell motility

- Genetic modifications can induce the fluourescence of individual proteins within a cell:
  - actin (red)
  - tubulin (green)
  - bead markers (blue)

RGB composite　　Visual grayscale



actin　　tubulin　　beads

Principal orthogonal modes

- The overall behavior of the cell can be modeled as the time (τ) variation of concentration of actin ($\boldsymbol{a}$), tubulin ($\boldsymbol{t}$), and other components of interest, e.g.,

$$\boldsymbol{D}\dot{\boldsymbol{a}}+\boldsymbol{K}\boldsymbol{a}=\boldsymbol{f}$$

  with $\boldsymbol{a}\in\mathbb{R}^m$, $m=hw=124,609$, $h=353, w=353$ = image height, width in pixels

- We seek a simpler description that highlights how changes in actin concentration occur in a correlated fashion in the entire cell

- Form a matrix of observed deviation of actin concentrations from the mean $\bar{\boldsymbol{a}}$ at p times

$$\boldsymbol{A}=\begin{pmatrix} \boldsymbol{a}_1-\bar{\boldsymbol{a}} & \boldsymbol{a}_2-\bar{\boldsymbol{a}} & \dots & \boldsymbol{a}_p-\bar{\boldsymbol{a}} \end{pmatrix}\in\mathbb{R}^{m\times p}$$

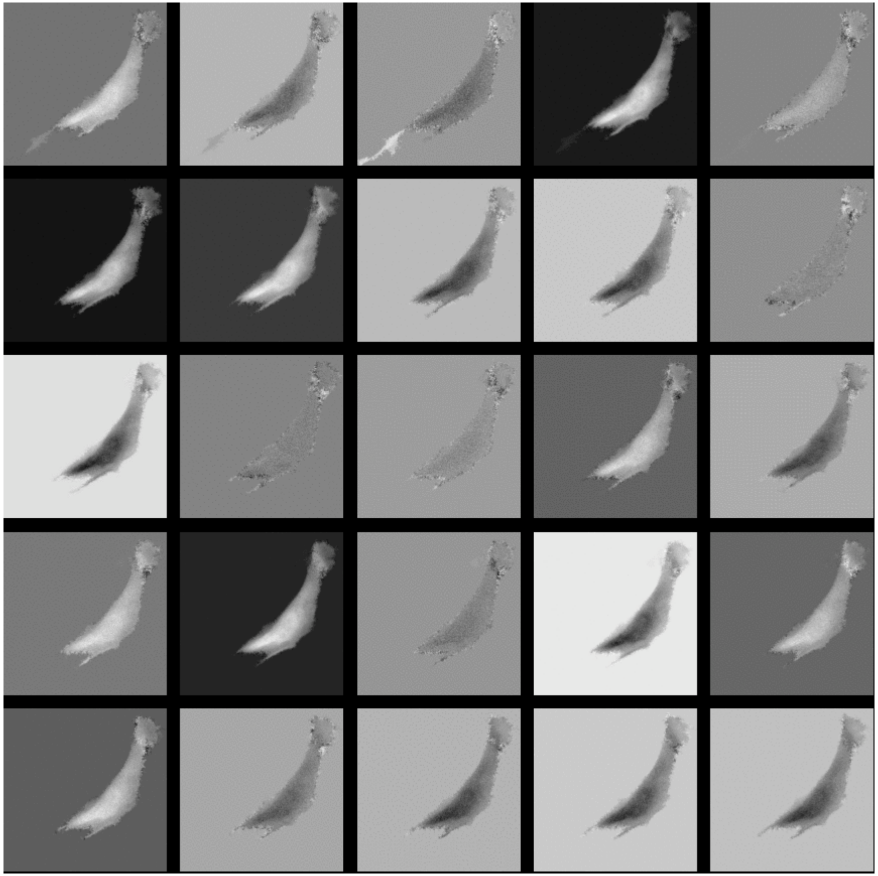
- The correlation between actin concentration deviations from mean is

$$\boldsymbol{C}=\boldsymbol{A}\boldsymbol{A}^T\in\mathbb{R}^{m\times m}$$

- The eigenvectors of $\boldsymbol{C}$ are orthogonal (symmetric matrix) and equal to left singular vectors of $\boldsymbol{A}$

$$\boldsymbol{C}=\boldsymbol{U}\boldsymbol{\Lambda}\boldsymbol{U}^T, \boldsymbol{A}=\boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T$$

- The principal modes of actin concentration are rendered below



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octave> cd /home/student/courses/MATH547/lessons/CellMotility;
octave> load data;
octave> [m,p]=size(data); disp([m p]);
```

124609 20

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octave> abar=mean(data,2); A = data - abar*ones(1,p);
octave> [U,S,V]=svd(A,'econ');
octave> disp(log10(diag(S(1:4,1:4)))');
```

-0.40778 -16.94721 -17.60992 -32.97520

```
octave> h=353; w=353; mode1=resize(U(:,2),h,w); mode2=resize(U(:,3),h,w);
octave> im1=0.5*(mode1+ones(h,w)); im2=0.5*(mode2+ones(h,w));
octave> imwrite(im1,'mode1.png'); imwrite(im2,'mode2.png');
```