1. MATH547 HOMEWORK 1

Topic:	Math@UNC environment
Post date:	May 14, 2020
Due date:	May 15, 2020

1.1. Background

This homework is meant as a tutorial on matrix computations, in particular:

- vector addition and scaling;
- vector norms;
- inner products;
- matrix-vector products;
- matrix-matrix products.

1.2. Theoretical questions

1.2.1. Vector addition and scaling

Problem. Consider n random walks on the real line with m unit steps starting from the origin. What is the mean distance and mean squared distance from the origin?

Answer.

1.2.2. Norms in E_m

Problem. Show that for $a_1, a_2 > 0$, $f: \mathbb{R}^2 \to \mathbb{R}_+$ defined by

$$f(\boldsymbol{x}) = a_1 x_1^2 + a_2 x_2^2,$$

is a norm in E_2.

Answer.

1.2.3. Norms in E_m

Problem. Show that for $a_1, a_2 > 0$, $f: \mathbb{R}^2 \to \mathbb{R}_+$ defined by

$$f(\boldsymbol{x}) = a_1 x_1^2 - a_2 x_2^2$$

is not a norm in E_2.

Answer.

1.2.4. Inner product in E_m

Problem. Show that for $a_1, a_2 > 0$, $s: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ defined by

$$s(\boldsymbol{x}, \boldsymbol{y}) = a_1 x_1 y_1 + a_2 x_2 y_2,$$

is a scalar product in E_2.

Answer.

1.2.5. Inner product in E_m

Problem. Show that for $a_1, a_2 > 0$, $s: \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$ defined by

$$s(\boldsymbol{x}, \boldsymbol{y}) = a_1 x_1 y_1 - a_2 x_2 y_2,$$

is not a scalar product in E_2.

Answer.

1.2.6. Matrix-matrix products

Problem. Find examples of matrices for which $\boldsymbol{AB} = \boldsymbol{BA}$, and $\boldsymbol{CD} \neq \boldsymbol{DC}$.

Answer.

1.3. Linear functionals and mappings in analysis of EEG data

Electroencephalograms (EEGs) are recordings of the electric potential on cranium skin. Research on brain activity uses EEGs to determine specific activity patterns in the brain. For example, epileptic seizures have a distinctive EEG signature. EEG data can be loaded from the course data directory.

```
octave] load /home/student/courses/MATH547ML/data/eeg/eeg;
```

```
octave] data=EEG.data'; [m n]=size(data); disp([m n]);
```

```
   30504      32
```

There are $n = 32$ sensor recordings with $m = 30504$ components each. A first common operation is scaling of the data.

```
octave] pdata=data./max(data)+meshgrid(0:n-1,0:m-1);
```

```
octave] hold on;
        for j=1:n
          plot(pdata(:,j));
        end;
        hold off;
```

```
octave] cd /home/student/courses/MATH547ML
```

```
octave] mkdir homework; cd homework
```

```
octave] mkdir hw01; cd hw01
```

```
octave] print -deps eeg.eps;
```

```
GL2PS info: OpenGL feedback buffer overflow
warning: gl2ps_renderer::draw: retrying with buffer size: 8.4E+06 B
GL2PS info: OpenGL feedback buffer overflow
warning: gl2ps_renderer::draw: retrying with buffer size: 1.7E+07 B
```

```
octave]
```

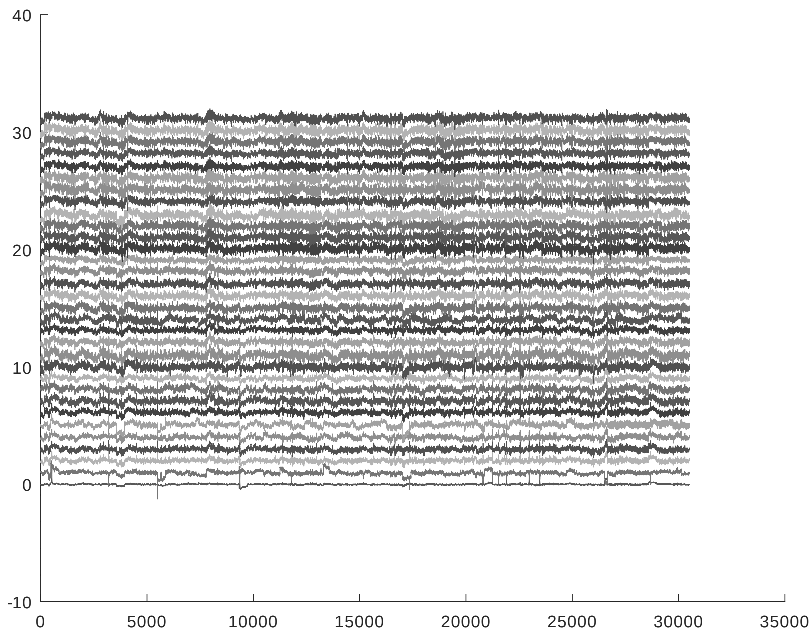

Figure 1.

1.3.1. Functional relationships in the data

Over some time subinterval can the data from a sensor be expressed as a function of data from another sensor?

1.3.2. Data magnitude (norm)

Partition sensor data into time subintervals. Over each subinterval, evaluate the p-norm of centered data, for $p = 1, 2$, 3, $p \to \infty$. Plot the p-norms over the entire time history.

1.3.3. Orthogonality of sensor data (inner product)

Over some time interval, determine the angle between sensor data. Is sensor data orthogonal? Plot the angle between sensor data over the entire time history.

1.3.4. Periodic representation of data (linear mapping)

Use template from Lesson01 to construct a representation of the data through periodic functions.