DATA RESONANCE

1. Bases induced by eigenmodes

The trigonometric functions $\{1, \cos t, \sin t, \cos 2t, \sin 2t, \ldots\}$ have been introduced as a particularly appropriate basis for periodic functions. The functions $\cos(kt), \sin(kt)$ are also known as solution of the homogeneous differential equation

$$y'' + k^2 y = 0.$$

The diferential operator is a linear mapping

$$\frac{\mathrm{d}^q}{\mathrm{d}t^q}(\alpha y + \beta z) = \alpha \frac{\mathrm{d}^q y}{\mathrm{d}t^q} + \beta \frac{\mathrm{d}^q z}{\mathrm{d}t^q},$$

and hence has an associated linear mapping. An approximation of the second-order differentiation operation is given by the finite difference formulas

$$y_i'' = y''(t_i) \cong \frac{1}{h^2}(y_{i+1} - 2y_i + y_{i-1})$$