Overview

- Quantitites
- Vectors
- Matrices
- Linear algebra problems

Numbers in mathematics

$\mathbb{N}$. The set of natural numbers, $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$, infinite and countable, $\mathbb{N}_+ = \{1, 2, 3, \ldots\}$;

$\mathbb{Z}$. The set of integers, $\mathbb{Z} = \{0, \pm 1, \pm 2, \pm 3, \ldots\}$, infinite and countable;

$\mathbb{Q}$. The set of rational numbers $\mathbb{Q} = \{p/q, p \in \mathbb{Z}, q \in \mathbb{N}_+\}$, infinite and countable;

$\mathbb{R}$. The set of real numbers, infinite, not countable, can be ordered;

$\mathbb{C}$. The set of complex numbers, $\mathbb{C} = \{x + iy, x, y \in \mathbb{R}\}$, infinite, not countable, cannot be ordered.

Numbers on a computer

Subsets of $\mathbb{N}$. The number types `uint8`, `uint16`, `uint32`, `uint64` represent subsets of the natural numbers (unsigned integers) using 8, 16, 32, 64 bits respectively.

Subsets of $\mathbb{Z}$. The number types `int8`, `int16`, `int32`, `int64` represent subsets of the integers. One bit is used to store the sign of the number.

Subsets of $\mathbb{Q}, \mathbb{R}, \mathbb{C}$. Computers approximate the real numbers through the set $\mathbb{F}$ of *floating point numbers*. Floating point numbers that use $b = 32$ bits are known as *single precision*, while those that use $b = 64$ are *double precision*.

Addition rules for	$\forall \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in V$
$\boldsymbol{a}+\boldsymbol{b} \in V$	Closure
$\boldsymbol{a}+(\boldsymbol{b}+\boldsymbol{c})=(\boldsymbol{a}+\boldsymbol{b})+\boldsymbol{c}$	Associativity
$\boldsymbol{a}+\boldsymbol{b}=\boldsymbol{b}+\boldsymbol{a}$	Commutativity
$\boldsymbol{0}+\boldsymbol{a}=\boldsymbol{a}$	Zero vector
$\boldsymbol{a}+(-\boldsymbol{a})=\boldsymbol{0}$	Additive inverse
Scaling rules for	$\forall \boldsymbol{a}, \boldsymbol{b} \in V$, $\forall x, y \in S$
$x\boldsymbol{a} \in V$	Closure
$x(\boldsymbol{a}+\boldsymbol{b})=x\boldsymbol{a}+x\boldsymbol{b}$	Distributivity
$(x+y)\boldsymbol{a}=x\boldsymbol{a}+y\boldsymbol{a}$	Distributivity
$x(y\boldsymbol{a})=(xy)\boldsymbol{a}$	Composition
$1\boldsymbol{a}=\boldsymbol{a}$	Scalar identity

Table 1. Vector space $\mathcal{V}=(V, S, +, \cdot)$ properties for arbitrary $\boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c} \in V$

- Real space, $\mathcal{R}_m=(\mathbb{R}^m, \mathbb{R}, +, \cdot)$

$$\boldsymbol{u}+\boldsymbol{v}=\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}+\begin{bmatrix} v_1 \\ \vdots \\ v_m \end{bmatrix}=\begin{bmatrix} u_1+v_1 \\ \vdots \\ u_m+v_m \end{bmatrix}, a\boldsymbol{u}=a\begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}=\begin{bmatrix} a u_1 \\ \vdots \\ a u_m \end{bmatrix}. \quad (1)$$

- Continuous functions, $\mathcal{C}^0=(C(\mathbb{R}), \mathbb{R}, +, \cdot)$, $(a f)(t)=a f(t)$, $(f+g)(t)=f(t)+g(t)$.

- Matrices are groupings of vectors $\boldsymbol{A} = [\ \boldsymbol{a}_1 \quad \boldsymbol{a}_2 \quad \dots \quad \boldsymbol{a}_n\ ], \boldsymbol{a}_1, \boldsymbol{a}_2, \dots, \boldsymbol{a}_n \in V$, $\mathcal{V} = (V, S, +, \cdot)$
- Vectors in $\mathcal{R}_m$ are given using the identity matrix $\boldsymbol{I}$, $\boldsymbol{x} \in \mathcal{R}_m$, $\boldsymbol{x} = \boldsymbol{I}\boldsymbol{x}$

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = x_1 \boldsymbol{e}_1 + x_2 \boldsymbol{e}_2 + \cdots + x_m \boldsymbol{e}_m,\ \boldsymbol{e}_1 = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \boldsymbol{e}_2 = \begin{bmatrix} 0 \\ 1 \\ \vdots \\ 0 \\ 0 \end{bmatrix}, \dots, \boldsymbol{e}_m = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}.$$

$$\boldsymbol{I} = [\ \boldsymbol{e}_1 \quad \boldsymbol{e}_2 \quad \dots \quad \boldsymbol{e}_m\ ].$$

- A linear combination is

$$\boldsymbol{b} = x_1\,\boldsymbol{a}_1 + x_2\,\boldsymbol{a}_2 + \ldots x_n\,\boldsymbol{a}_n$$

- The matrix-vector product is defined to represent linear combinations

$$\boldsymbol{A} = [\ \boldsymbol{a}_1 \quad \boldsymbol{a}_2 \quad \ldots \quad \boldsymbol{a}_n\ ], \boldsymbol{b} = \boldsymbol{A}\boldsymbol{x}$$

```
octave] ex=[1; 0]; ey=[0; 1];
octave] b=[0.2; 0.4]; I=[ex ey]; I*b
ans =

   0.20000
   0.40000

octave] th=pi/6; c=cos(th); s=sin(th);
octave] tvec=[c; s]; nvec=[-s; c];
octave] A=[tvec nvec];
octave] x=A\b
x =

   0.37321
   0.24641
```

```
octave] [x(1)*tvec x(2)*nvec]
ans =

   0.32321  -0.12321
   0.18660   0.21340

octave]
```

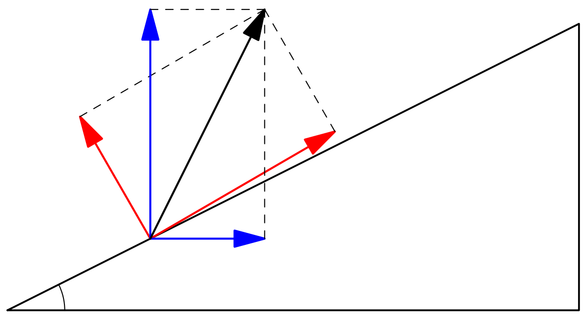

Figure 1. Alternative decompositions of force on inclined plane.

```
octave] m=1000; h=2*pi/m; j=1:m;
octave] t(j)=(j-1)*h; t=transpose(t);
octave] n=5; A=[];
octave] for k=1:n
          A = [A sin(k*t)];
        end
octave] bt=t.*(pi-t).*(2*pi-t);
octave] x=A\bt;
octave] b=A*x;
octave] s=50; i=1:s:m;
        ts=t(i); bs=bt(i);
        plot(ts,bs,'ok',t,b,'r');
octave] print -depsc L01Fig02.eps
```

```
octave] close;
octave]
```

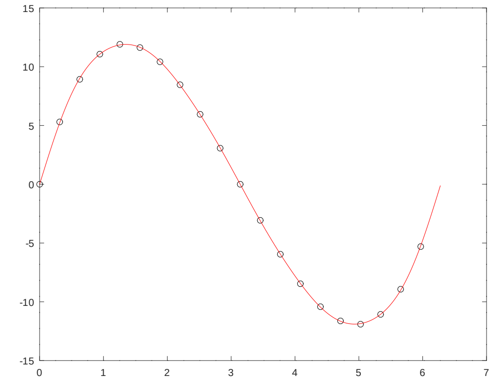

Figure 2. Comparison of least sqaures approximation (red line) with samples of exact function $b(t)$.