Definition. (Vector Subspace) . *$\mathcal{U} = (U, S, +, \cdot)$ with $U \neq \varnothing$ is a vector subspace of vector space $\mathcal{V} = (V, S, +, \cdot)$ over the same field of scalars S if $U \subseteq V$ and $\forall a, b \in S$, $\forall \boldsymbol{u}, \boldsymbol{v} \in U$, the linear combination $a\boldsymbol{u} + b\boldsymbol{v} \in U$.*

1. *Column space*, $C(\boldsymbol{A}) = \{\boldsymbol{b} \in \mathbb{R}^m | \, \exists \boldsymbol{x} \in \mathbb{R}^n \text{ such that } \boldsymbol{b} = \boldsymbol{A}\boldsymbol{x}\} \subseteq \mathbb{R}^m$, the part of $\mathbb{R}^m$ *reachable* by linear combination of columns of $\boldsymbol{A}$
2. *Left null space*, $N(\boldsymbol{A}^T) = \{\boldsymbol{y} \in \mathbb{R}^m | \, \boldsymbol{A}^T\boldsymbol{y} = 0\} \subseteq \mathbb{R}^m$, the part of $\mathbb{R}^m$ *not reachable* by linear combination of columns of $\boldsymbol{A}$
3. *Row space*, $R(\boldsymbol{A}) = C(\boldsymbol{A}^T) = \{\boldsymbol{c} \in \mathbb{R}^n | \, \exists \boldsymbol{y} \in \mathbb{R}^m \text{ such that } \boldsymbol{c} = \boldsymbol{A}^T\boldsymbol{y}\} \subseteq \mathbb{R}^n$, the part of $\mathbb{R}^n$ *reachable* by linear combination of rows of $\boldsymbol{A}$
4. *Null space*, $N(\boldsymbol{A}) = \{\boldsymbol{x} \in \mathbb{R}^n | \, \boldsymbol{A}\boldsymbol{x} = 0\} \subseteq \mathbb{R}^n$, the part of $\mathbb{R}^n$ *not reachable* by linear combination of rows of $\boldsymbol{A}$