Final Examination

Solve the following problems (5 course points each). Present a brief motivation of your modeling. This is an open-book examination. You are free to use course materials, information from websites, scientific journals or search engines.

How do starfish eat?

Problem statement. A starfish feeds by extending its cardiac stomach out of its mouth to cover digestible parts of its prey, such as mussels and clams. The mollusk's shell is pried open by the starfish arms. The prey tissue is partially digested and then drawn back into its pyloric stomach to complete digestion.

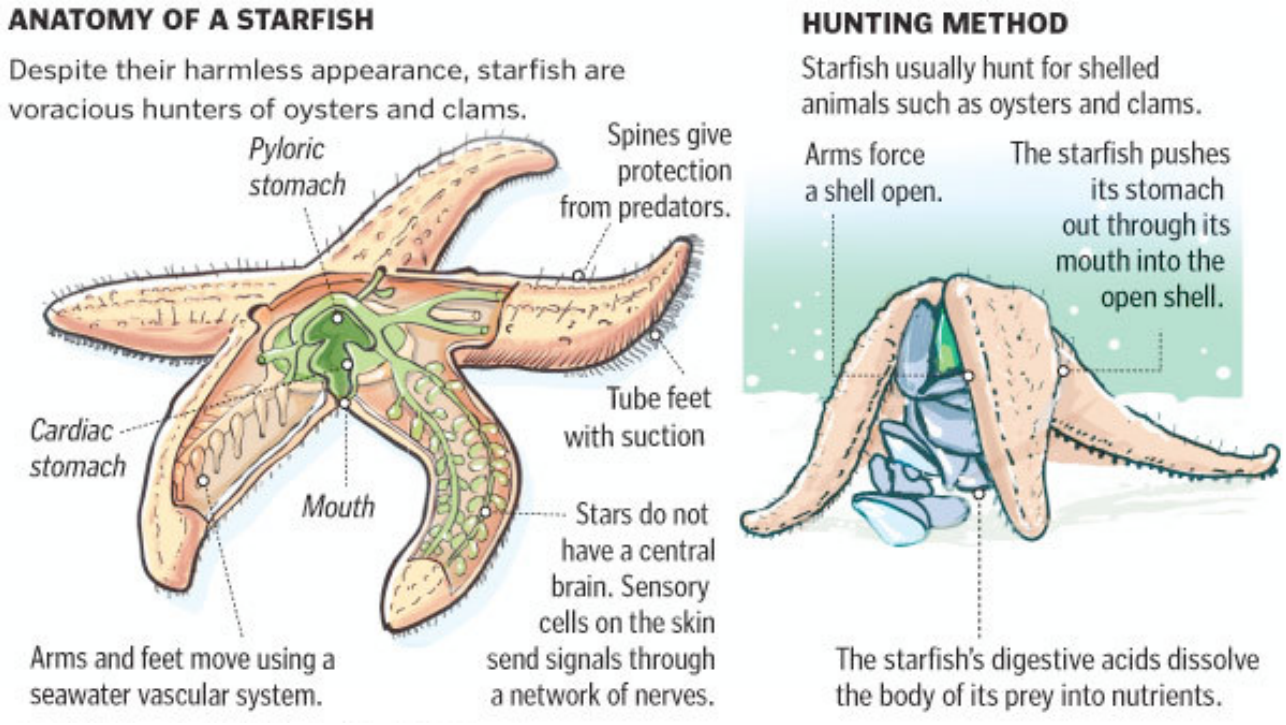

Figure 1. Schematic of starfish digestive system

A first approach to mathematical modeling of starfish feeding behavior is to consider overall energy requirements and link these to evolutionary adaptations of the starfish. The following four questions represent a first step in this process.

Question 1: Why move at all?. Light penetrates shallow ocean water and allows sustainment of phytoplankton with a caloric content of 4 kcal/g. A typical phytoplankton concentration is 10^6 unicellular organisms per liter of ocean water, with a phytoplankton cell having a typical size of 10 μm and density equal to that of ocean water (1.03 g/cm^3). A typical starfish has an external diameter of 25 cm and central diameter of 5 cm, arm length of 20 cm, arm cross-section diameter of 3 cm, and density of 1.06 g/cm^3. Starfish have a typical metabolic rate of 0.05 mg O_2 per hour, per gram of starfish tissue. Energy release in phosphorylation is approximately 5 calories per liter of oxygen consumed. Could a starfish maintain its metabolism by diffusive feeding through its mouth? Consider that phytoplankton have a diffusivity of 10^{-5} cm^2/s in ocean water. Evolutionary adaptation strongly implies that feeding by passive diffusion is not viable.

Question 2: How many starfish arms?. If passive diffusive feeding is not possible some alternative mechanism is required, for instance movement to cover a larger feeding area. Essentially movement increases the effective size of the starfish "mouth" to the area traversed by the central part of the starfish. Construct a random walk model of starfish motion. In each 5 minute long step the starfish moves in the direction of an arm by half of the arm length. In the next step it randomly chooses a different arm direction. As a result of this motion its mouth covers a certain area of the ocean bottom. Construct a model of the area traversed by the starfish as a function of the number of arms and the arm length. Present a mathematical argument against "two-armed" starfish that confirms evolutionary adaptation. How many arms should a starfish have? Is movement sufficent to ensure adequate feeding?