Study of neuron models

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Abstract

This report describes the classic Hodgkin-Huxley model of neuron electrochemical activity that leads to synapses, extended to consideration of unidimensional propagation along a sequence of neurons.

1 Introduction

Multicellular biological organisms with differentiated cells need to establish information transfer between various specialized cells. We consider an extension of the classical Hodgkin-Huxley electrochemical neuron model [1] to series of neurons. The historical development of the Hodgkin-Huxley model [2] started with voltage measurements along the giant axon of the *Loligo forbesi* squid (Fig. 1,Left) leading to the discovery of active circuit elements across the axon membrane later identified as ion channels. The structure of ion channels has been extensively studied leading to the identification of the structural chemistry that reveals a central channel bounded by polymer components.

The identification of electrical pulse shape as a function of time $V(t)$ within an axon cross section was the first step in tracking pulse propagation along the axon's kength and further transmission to the dendrites of an adjacent neuron, the process known as synapse. In this study an extension to pulse propagation across multiple neurons is investigated. Neurons are considered linked in series, the first step in the more general problem of electrical signal propagation through a neural network with multiple connections and possible pathways.

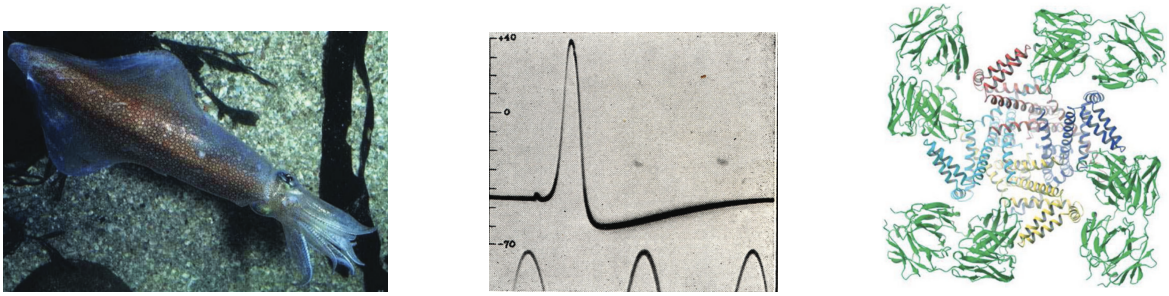

Figure 1. (Left) Squid; (Center) typical electrical pulse shape $V(t)$ from Hodkin-Huxley experiments (Right) Structural ion channel model from Nobel lecture by Roderick MacKinnon (2003) [2].

1.1 Neuron physiology

1.1.1 Neuron structure

In order to model electrical signal propagation a neuron structural model is required. As shown in Fig. 2, a neuron is a specialized cell with an elongated extension called an axon

and several shorter extensions called dendrites. In the model considered here electrical pulses travel along dendrites and axons at constant speed such that

$$V(x,t) = V(x - ct, t),$$

with the propagation velocity c determined by the detailed biochemistry of ion channel opening and closing.

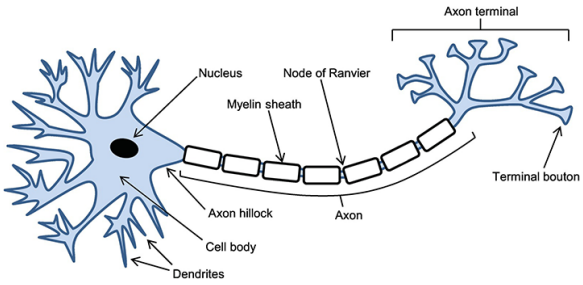

Figure 2. Basic structure of a neuron.

1.1.2 Electrochemistry of the neuron

While the detailed molecular dynamics of ion channels (Fig. 1, Right) is a complex problem, the time dependence of voltage in an axon cross-section can be directly related to concetrations of a few significant ions, namely K^+, Na^+, Cl^- and electrical properties of the axon membrane. Neuron membranes exhibit polarization achieved by: passive ion transport and active ion transport mediated by Na/K ion channel pumps. A brief overview of the key physical parameters includes:

- a resting polarization voltage across an axon membrane of $V_r = -70$ mV;
- the membrane capacitance is $C_m = 0.001$ (F/cm^2)
- resting voltages for the individual ions are $E_{\text{Na}} = 55$ mV, $E_K = -82$ mV, $E_\ell = -59$ mV
- ion conductance constants are $\bar{g}_{\text{Na}} = 70.7$ (m-mhos/cm^2), $\bar{g}_K = 24.34$ (m-mhos/cm^2), $g_\ell = 0.3$ (m-mhos/cm^2).

1.2 Hodgkin-Huxley model

1.2.1 Equivalent electrical circuit

The Hodgkin-Huxley experiments suggested an equivalent electrical model (Fig. 3) for the axon cross-section consisting of a capacitator C_m, voltage-dependent conductors g_n, g_L governing the electrical current from inside the axon to the extracellular medium.

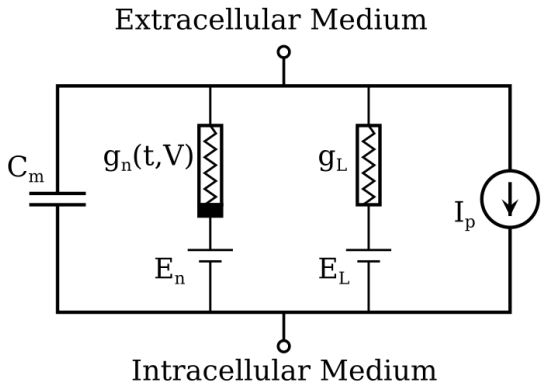

Figure 3. In the Hodgkin-Huxley model of a neuron, an axon cross section is modeled by a parallel electrical circuit comprising three conductances (the K, Na, Cl ion channels) and a capacitator (the membrane).

1.2.2 Ion transport across neuron membrane

Assuming constant ion concentrations in an axon cross-section the rate of change of K^+, Na^+, Cl^- concentrations is given by the ODE system.

$$\frac{\mathrm{d}m}{\mathrm{d}t} = \alpha_m(V)(1-m) - \beta_m(V)m \tag{1}$$

$$\frac{\mathrm{d}n}{\mathrm{d}t} = \alpha_n(V)(1-n) - \beta_n(V)n \tag{2}$$

$$\frac{\mathrm{d}h}{\mathrm{d}t} = \alpha_h(V)(1-h) - \beta_h(V)h \tag{3}$$

The equations express a two time scale concentration evolution between values of 0 and 1 with time scales given by α_i, β_i respectively (Fig.)

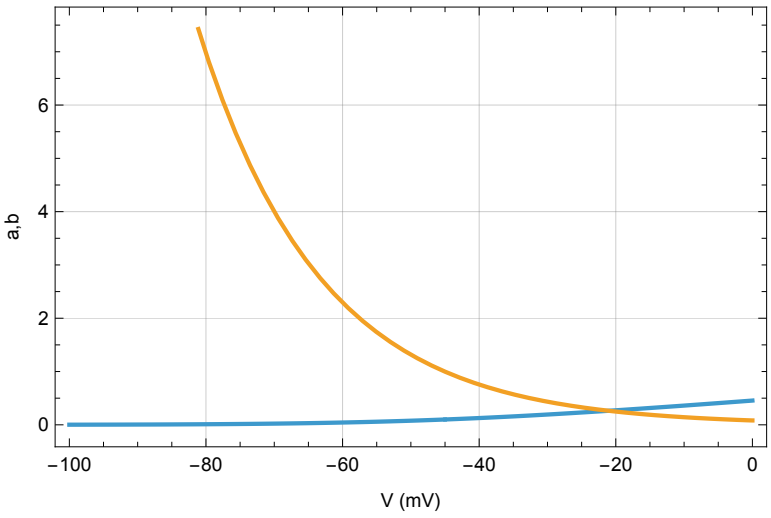

Figure 4.

1.2.3 Current balance through membrane

Ions are transported across the axon membrane due to a voltage difference. Transport occurs

across ion channels, both passively and actively. The overall rate of change of voltage is given by

- $$C_m \frac{\mathrm{d}V}{\mathrm{d}t} = \bar{g}_{\mathrm{Na}} m^3 h \left(V - E_{\mathrm{Na}}\right) + \bar{g}_{\mathrm{K}} n^4 \left(V - E_{\mathrm{K}}\right) + g_\ell \left(V - E_\ell\right) + P(t) \tag{4}$$

```
In[66]:= iNa = gNa (m[t])^3 h[t] (V[t] - ENa);
         iK = gK (n[t])^4 (V[t] - EK);
         iCl = gl (V[t] - El);
```

```
In[73]:= ODE4 = Cm V'[t] == iNa + iK + iCl + P[t];
```

```
In[74]:=
```

1.3 Propagation of electrical impulses along neuron axon

1.3.1 Time variation within an axon cross-section

- Start from a set of initial conditions $m(0) = m_0$, $n(0) = n_0$, $h(0) = h_0$, $V(0) = V_0$.
- Numerical solution of the system of ordinary differential equations (1) to (4) with no forcing but from some perturbed initial state leads to typical behavior as shown in Fig.

```
In[74]:= ODEsys = {ODE1,ODE2,ODE3,ODE4};
```

```
In[75]:= IVP = Flatten[{ODEsys,IC}];
```

```
In[38]:= P[t_]=0;
```

```
In[51]:= HHsol[t_] = {m[t],n[t],h[t],V[t]}  /. NDSolve[IVP,{m[t],n[t],
         h[t],V[t]},{t,0,0.05}][[1,1]]
```

```
 NDSolve::deqn: Equation or list of equations expected instead of -70
in the first argument m'[t] == (-70 - V[t])/18 0.01 (1 - m[t]) (-45 -
V[t]) -4 E m[t] + ----------, 2.5 - 0.1 (70 + V[t]) -1 + E (-70 -
V[t])/80 0.01 (1 - n[t]) (-60 - V[t]) n'[t] == -0.125 E n[t] +
----------, 1 - 0.1 (70 + V[t]) -1 + E h'[t] == al[<<1>>] <<1>> +
<<1>>, <<4>>, -70.
ReplaceAll::reps: (-70 - V[t])/18 0.01 (1 - m[t]) (-45 - V[t]) m'[t]
== -4 E m[t] + ---------- 2.5 - 0.1 (70 + V[t]) -1 + E is neither a
list of replacement rules nor a valid dispatch table, and so cannot be
used for replacing.
```

$$\{m(t), n(t), h(t), V(t)\} /. m'(t) = \frac{0.01\,(1 - m(t))\,(-V(t) - 45)}{e^{2.5 - 0.1(V(t)+70)} - 1} - 4\,m(t)\,e^{\frac{1}{18}(-V(t)-70)}$$

```
In[81]:= NDSolve[IVP,{m[t],n[t],h[t],V[t]},{t,0,0.05}][[1,1]]
```

```
 NDSolve::deqn: Equation or list of equations expected instead of -70
in the first argument m'[t] == (-70 - V[t])/18 0.01 (1 - m[t]) (-45 -
V[t]) -4 E m[t] + ----------, 2.5 - 0.1 (70 + V[t]) -1 + E (-70 -
V[t])/80 0.01 (1 - n[t]) (-60 - V[t]) n'[t] == -0.125 E n[t] +
----------, 1 - 0.1 (70 + V[t]) -1 + E 0.07 (1 - h[t]) h'[t] == ------
+ <<1>>, <<4>>, -70. 0.05 (70 + V[t]) E
```

$$m'(t) = \frac{0.01\,(1-m(t))\,(-V(t)-45)}{e^{2.5-0.1(V(t)+70)}-1} - 4\,m(t)\,e^{\frac{1}{18}(-V(t)-70)}$$

```
In[80]:= IVP[[8]]
```

$$-70$$

```
In[81]:=
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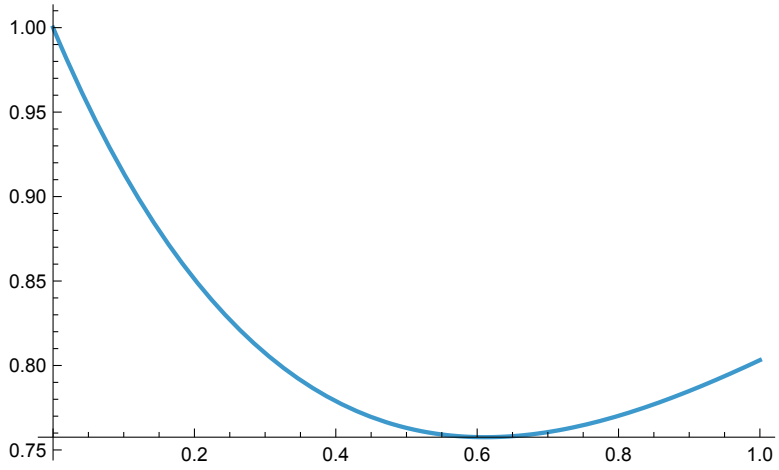

Figure 5.

2 Methods

2.1 Change of ion concentration in axon cross-section

2.2 Membrane voltage

2.3 Driving force

3 Results

4 Conclusions

Bibliography

[1] Mathematical Biology: An Introduction with Maple and Matlab | SpringerLink. https://link.springer.com/book/10.1007/978-0-387-70984-0.

[2] Christof J. Schwiening. A brief historical perspective: Hodgkin and Huxley. *The Journal of Physiology*, 590(11):2571–2575, 2012.