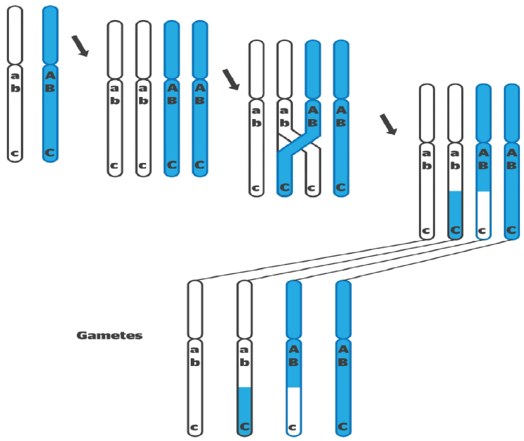
- Biology review:
  - Evolution
  - Cells
  - Reproduction
- Population models:
  - Exponential growth (Malthus)
  - Maturation delay

Genetic information can be reproduced:
- from one individual (asexual)
- from two individuals (sexual)



Crossing-over and recombination during meiosis

During meiosis a diploid cell:

1. Duplicates (Metaphase 1)
2. Possibly has gene crossover
3. Separates into 4 gametes (haploid)

- Reproduction leads to genetic variation
- Non-competitive genes are not propagated, competitive genes proliferate (cf. Dawkins, *The Selfish Gene*)
  - Selection is due to scarcity of resources: space, energy, materials
  - Death of non-competitive genetic variations before they can reproduce is a significant feature
- Sufficient genetic differentiation leads to new species

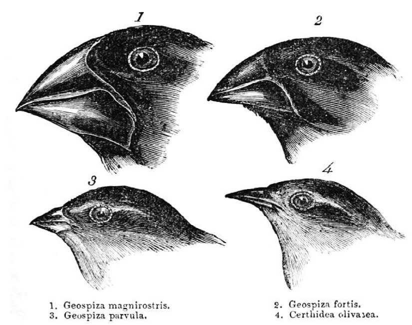


Figure 1. Galapagos Islands finches

- Sexual reproduction though to have occurred ~ 2 BYA, Proterozoic ("early life") eon, after Hadean ("Hell-like") eon and Archean ("origins") eon, Low O_2, high CO_2, CH_4 concentrations

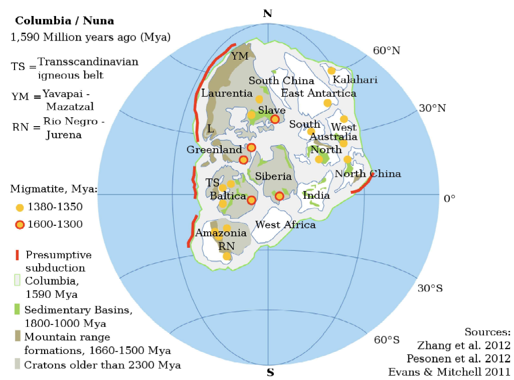


Figure 2. Earth in early Proterozoic, Columbia supercontinent ~ 1.6 BYA

- Sexual reproduction enables large variability in genetic encoding, setting stage for Cambrian explosion (~ 540 MYA), as seen in

$$\text{Species} \in \text{Genus} \in \text{Family} \in \text{Order} \in \text{Class} \in \text{Phylum} \in \text{Kingdom} \in \text{Domain}$$

- Prokaryote cells (bacteria, algae) mostly lack internal membranes (e.g., nuclei)
- Eukaryote cells exhibit multiple organelles

$$\mathrm{Eukarya} = \{\mathrm{Plantae}, \mathrm{Protozoa}, \mathrm{Animalia}, \mathrm{Chromista}, \mathrm{Fungi}\}$$

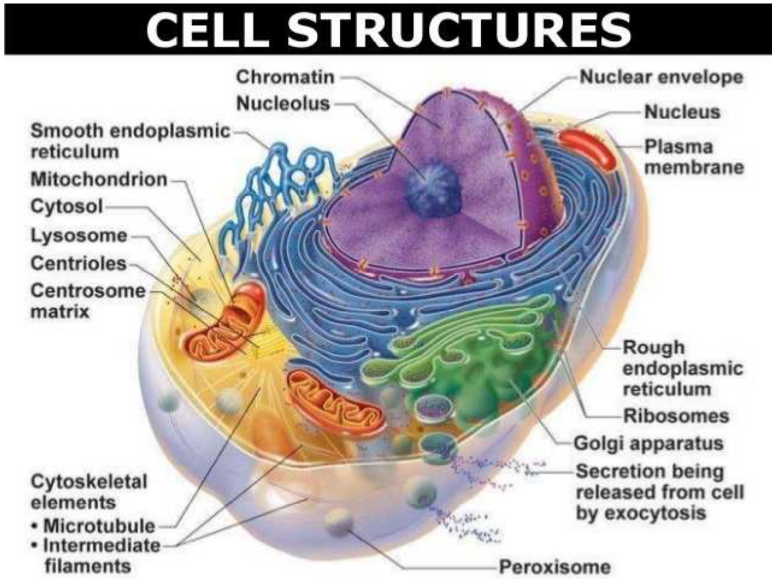


Figure 3. Typical eukaryote cell structure, the product of billion of years of randomness in sexual reproduction coupled with survival of the fittest

- Reproduction induced by growth?
  - Favorable environmental conditions leads to growth, volume $V \sim R^3$ increases with cell radius R
  - Waste elimination, nutrient uptake occurs through surface $S \sim R^2$
  - $V/S \sim R$ increases, to point where division is preferable to further growth?
- Unconstrained population growth at population growth rate

$$r = \frac{\text{birth rate} - \text{death rate}}{\text{population size}} > 0,$$

leads to (continuum Malthus model) $y' = ry \Rightarrow y(t) = e^{rt} y(0)$

or to (discrete Malthus model) $y_{t+1} - y_t = \rho y_t \Rightarrow y_t = (1+\rho)^t y_0$

- Maturation delay τ: $y'(t) = r y(t-\tau)$, initial condition $y_0(-\tau, 0)$ known

```
In[3]:= z[t_,r_] = y[t] /. DSolve[{y'[t] == r y[t-1],y[t /; t<=0]==1},y,{t,0,
        10}][[1,1]];
        TableForm[Table[{r,z[10,r]},{r,0.1,1.0,0.1}],TableHeadings->{{},{"r",
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```

r	z1[10,r]
0.1	2.50101
0.2	5.4849
0.3	10.9333
0.4	20.2603
0.5	35.4344
0.6	59.1225
0.7	94.8556
0.8	147.22
0.9	222.073
1.	326.791

- $y'(t) = ry(t-\tau), r = 1, \tau = 1$. Initial condition $y(t) = e^{t/10}$, $-1 \leqslant t \leqslant 0$
- Solve on [0,1]

```
In[6]:= f0[t_]=Exp[t/10];
        f1[t_]=y[t]/. DSolve[{y'[t]==f0[t-1],y[0]==1},y[t],t][[1,1]]
```

- Solve on [1,2]

```
In[7]:= f2[t_]=y[t]/.DSolve[{y'[t]==f1[t-1],y[1]==f1[1]},y[t],t][[1,1]]
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$$\frac{10\,e^{t/10} - 10 + \sqrt[10]{e}}{\sqrt[10]{e}}$$

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```

$$\frac{\sqrt[5]{e}\,t - 10\,\sqrt[10]{e}\,t + 100\,e^{t/10} + 10\,\sqrt[5]{e} - 100\,\sqrt[10]{e}}{\sqrt[5]{e}}$$

- What happens after seven years of plenty or seven years of scarcity?

Define models with time histories $e^{-t}, e^0 = 1, e^t, 1 + \frac{1}{4}\sin(2\pi t)$

```
In[8]:= z1[t_,r_] = y[t] /. DSolve[{y'[t] == r y[t-1],y[t /; t<=0]==Exp[-t]},y,{t,0,
        7}][[1,1]];
        z2[t_,r_] = y[t] /. DSolve[{y'[t] == r y[t-1],y[t /; t<=0]==1},y,{t,0,
        7}][[1,1]];
        z3[t_,r_] = y[t] /. DSolve[{y'[t] == r y[t-1],y[t /; t<=0]==Exp[t]},y,{t,0,
        7}][[1,1]];
        z4[t_,r_] = y[t] /. DSolve[{y'[t] == r y[t-1],y[t /;
        t<=0]==1+0.25*Sin[2*Pi*t]},y,{t,0,7}][[1,1]];
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```

Null

- Interaction of growth rate and history upon population after 7 years

```
In[9]:= TableForm[Table[N[{r,z1[7,r],z2[7,r],z3[7,r],z4[7,r]}],{r,0.1,1.0,
        0.1}],TableHeadings->{{},{"r","7LeanYears","7FlatYears","7FatYears",
        "7Up&DownYears"}}]

In[10]:=
```

- Interaction of growth rate and history upon population after 7 years

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In[9]:= TableForm[Table[N[{r,z1[7,r],z2[7,r],z3[7,r],z4[7,r]}],{r,0.1,1.0,
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```

r	7LeanYears	7FlatYears	7FatYears	7Up&DownYears
0.1	2.02323	1.90192	1.8401	1.90252
0.2	3.68552	3.30439	3.11098	3.30784
0.3	6.22552	5.37389	4.94328	5.38454
0.4	9.93124	8.3075	7.48914	8.3327
0.5	15.1448	12.3355	10.9237	12.3865
0.6	22.2673	17.724	15.4467	17.8173
0.7	31.7639	24.7777	21.2843	24.9361
0.8	44.1689	33.8426	28.6904	34.0967
0.9	60.091	45.309	37.9486	45.6991
1.	80.2186	59.6141	49.3739	60.1917

```
In[10]:=
```