- Fundamental solution
- Diffusion of momentum and viscous flow

- Diffusion equation $\partial c / \partial t = \alpha \partial^2 c / \partial x^2$
- Rewrite as an operator applied to the dependent variable

$$\mathcal{D} c = 0, \mathcal{D} = \frac{\partial}{\partial t} - \alpha \frac{\partial^2}{\partial x^2}$$

- Similar to Fourier, Laplace transforms $\mathcal{F} \mathcal{L} \mathcal{D} c = 0 \Rightarrow i\omega \hat{c} + \alpha k^2 \hat{c} = (i\omega + k^2)\hat{c} = 0$
- The solution to $\mathcal{D} c = \delta$ (Dirac delta) is called the fundamental solution G

$$G = \frac{1}{\sqrt{2\pi\alpha t}} \mathrm{Exp}\left[-\frac{x^2}{4\alpha t} \right], \sigma^2 = 2\alpha t$$

- The solution to $\mathcal{D} c = f$ is $c = G * f = \iint G(t - \tau, x - y)\, f(\tau, y)\, \mathrm{d}\tau\, \mathrm{d}y$

- Spherical cell of radius a consumes O_2 at rate R per unit volume and is supplied by diffusion through surface of area A

$$\alpha A \frac{\mathrm{d}c}{\mathrm{d}r} = VR \Rightarrow \frac{\mathrm{d}c}{\mathrm{d}r} = \frac{Rr}{3\alpha}$$

- Concetration of O_2 at center of sphere is zero, and C_{O_2} at $r = a$

$$\int_0^{C_{O_2}} \mathrm{d}c = \int_0^a \frac{Rr}{3\alpha} \mathrm{d}r \Rightarrow C_{O_2} = \frac{Ra^2}{6\alpha}$$

- Biological values $R = 0.3$ μl/g/s, $C_{O_2} = 7$ μl/cm^3, $\alpha = 2 \times 10^{-5}$ cm/s

$$a^2 \cong 0.003\,\mathrm{cm}^2 \Rightarrow a \cong 0.5\,\mathrm{mm}$$

- Fick's law describes diffusive transport of mass $J = -\alpha \partial c / \partial x$
- Microscopic diffusion transports momentum and energy in addition to mass:
  - $\rightarrow$ momentum diffusion: Stokes' law $\tau = \mu \partial u / \partial y = \rho \nu \partial u / \partial y = \nu \partial(\rho u) / \partial y$
  - $\rightarrow$ energy diffusion: Fourier's law $q = -k \partial T / \partial x$
- Resistance to flow in a pipe

$$\text{Driving force} = \Delta p \pi r^2 = \tau 2\pi r l = \text{Friction force}$$

$$\Delta p \pi r^2 = -\mu \frac{\mathrm{d}u}{\mathrm{d}r} 2\pi r l \Rightarrow u(r) = -\frac{\Delta p}{4 l \mu} r^2 + C$$

$$\text{at } r = R, u(R) = 0 \Rightarrow C = \frac{\Delta p}{4 l \mu} R^2 \,(\text{Poiseuille flow}) \Rightarrow$$

$$u(r) = \frac{\Delta p}{4 l \mu}(R^2 - r^2), \, Q = \int_0^R u(r)\, \mathrm{d}S = \frac{\pi \Delta p R^4}{8 l \mu}$$