



- Random walks
- Diffusion equation



- The atomic hypothesis: matter is composed of small individual particles in perpetual motion that interact with one another, attracting one another when distant, repelling one another when close (R. Feynman)
- The state of perpetual motion at the atomic scale is perceived at a macroscopic scale as temperature, and the average kinetic energy per atomic degree of freedom is  $kT/2$ ,  $k = 1.38 \times 10^{-23}$  J/K. An atom with mass  $m$  and average velocity  $\bar{v}$  has kinetic energy at temperature  $T$

$$\frac{m \bar{v}^2}{2} = 3 \frac{kT}{2}$$

- The average microscopic velocities of various atoms, molecules can be found as

$$\bar{v}^2 = \frac{3kT}{m}$$



- Consider a particle that can take position  $x \in \mathbb{Z}$
- Let  $p(m, n)$  denote that particle starting from  $x = 0$  is at position  $m$  after  $n$  steps
- Denote by  $r, l$  number of steps to the right (+) and left (−)

$$m = r - l, n = r + l \Rightarrow r = \frac{1}{2}(m + n), l = \frac{1}{2}(n - m)$$

- Toss a coin  $n$  times, there are  $2^n$  possible results of which

$$C(n, r) = \frac{n!}{r! (n - r)!}$$

- Probability of being at position  $m$  is  $p(m, n) = C(n, r) / 2^n$