- Chemical kinetics
- Enzyme kinetics

- Consider $A+B \rightarrow X+Y$

$$\frac{\mathrm{d}X}{\mathrm{d}t}=\frac{\mathrm{d}Y}{\mathrm{d}t}=-\frac{\mathrm{d}A}{\mathrm{d}t}=-\frac{\mathrm{d}B}{\mathrm{d}t}$$

- Denote concentration by brackets, lower case, $[A]=a=A/(\text{volume})$
- Law of mass action: $\text{rate}=k[A][B]=k\,a\,b$. Obtain nonlinear ODE

$$\frac{\mathrm{d}x}{\mathrm{d}t}=k\,a\,b=k\,(a_0+x_0-x)(b_0+x_0-x) \Rightarrow$$

$$\left[\frac{1}{a_0+x_0-x}-\frac{1}{b_0+x_0-x}\right]\mathrm{d}x=(b_0-a_0)\,k\,\mathrm{d}t$$

- Stationary points $x=a_0+x_0$, $x=b_0+x_0$.

- Consider $A + B \leftrightarrows X + Y$

$$\frac{\mathrm{d}X}{\mathrm{d}t} = \frac{\mathrm{d}Y}{\mathrm{d}t} = -\frac{\mathrm{d}A}{\mathrm{d}t} = -\frac{\mathrm{d}B}{\mathrm{d}t}$$

- Denote concentration by brackets, lower case, $[A] = a = A/(\mathrm{volume})$
- Apply law of mass action for both directions

$$\frac{\mathrm{d}x}{\mathrm{d}t} = k_+ a b - k_- x y = k_+ (a_0 + x_0 - x)(b_0 + x_0 - x) - k_- x (y_0 - x_0 + x) \Rightarrow$$

- Stationary points: $x' = 0 \Rightarrow$ a quadratic equation with roots α, β
- Rewrite ODE as $x' = (k_+ - k_-)(x - \alpha)(x - \beta)$

- Consider $S+E\leftrightarrows C$, and $C\rightarrow P+E$

$$\frac{\mathrm{d}C}{\mathrm{d}t}=-\frac{\mathrm{d}S}{\mathrm{d}t}=-\frac{\mathrm{d}E}{\mathrm{d}t},\frac{\mathrm{d}P}{\mathrm{d}t}=\frac{\mathrm{d}E}{\mathrm{d}t}=-\frac{\mathrm{d}C}{\mathrm{d}t}$$

- Apply law of mass action for both reactions

$$\begin{aligned} c' &= k_+\,e\,s-k_-\,c-k_2\,c\\ s' &= -k_+\,e\,s+k_-\,c\\ e' &= -k_+\,e\,s+k_-c+k_2\,c\\ p' &= k_2\,c \end{aligned}$$

- Stationary points: solution of a nonlinear system
- Eliminate e, p with $c_0=0$

$$c'=k_+\,s(e_0-c)-(k_-+k_2)c, s'=-k_+s\,(e_0-c)+k_-c$$