## 1 Growth

$$\begin{array}{ll} \text{With} & \text{Without} \\ V_1 & \frac{4\pi}{3 \times 2^k} R^3 = \frac{4\pi}{3} \left(\frac{R}{2^{k/3}}\right)^3 & \frac{4\pi}{3} R^3 \\ V & \frac{4\pi}{3} \left(\frac{R}{2^{k/3}}\right)^3 \times 2^k = \frac{4\pi}{3} R^3 & \frac{4\pi}{3} R^3 \\ S_1 & 4\pi \left(\frac{R}{2^{k/3}}\right)^2 & 4\pi R^2 \\ \text{S} & 4\pi \left(\frac{R}{2^{k/3}}\right)^2 \times 2^k = 4\pi R^2 2^{k/3} & 4\pi R^2 \end{array}$$

Table 1. Growth with/without reproduction  $2^k$  offspring. Offspring have the same overall volume as the progenitor.

## 2 Population models

$$y' = ry \Rightarrow y(t) = e^{rt} y(0) = \left(1 + rt + \frac{1}{2}(rt)^2 + \dots + \right) y_0$$
$$y_{t+1} - y_t = \rho y_t \Rightarrow y_t = (1+\rho)^t y_0 = \left(1 + \rho t + \frac{t(t-1)}{2}\rho^2 + \dots + \right) y_0$$

## 2.1 Ordinary versus delay differential equations

• Ordinary differential equation

IVP: 
$$y'(t) = f(t, y(t)), y(0) = y_0$$

• Delay differential equation

IVP: 
$$y' = f(t, y(t - \tau)), y(t) = f_0(t)$$
 for  $-\tau \le t \le 0$