Consider the PDE

$$\frac{\partial P}{\partial t} + \frac{\partial P}{\partial y} = -\mu(t, y)\, P \qquad (1)$$

A curve in the (y, t) plane

$$\Gamma\colon\, y(s), t(s)$$

Consider the restriction of $P(t, y)$ to the curve

$$P_\Gamma(t, y) = p(s) = P(t(s), y(s))$$

$$\frac{\mathrm{d}P}{\mathrm{d}s} = \frac{\partial P}{\partial t}\frac{\mathrm{d}t}{\mathrm{d}s} + \frac{\partial P}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}s}$$

Choose t', y' by identification with coefficients of (1)

$$\frac{\mathrm{d}t}{\mathrm{d}s} = 1, \frac{\mathrm{d}y}{\mathrm{d}s} = 1$$

Solving leads to the characteristic curves

$$y = t + C$$

Along the characteristic curves

$$\frac{\mathrm{d}P}{\mathrm{d}s} = -\mu(t, t + C)P(s)$$

$$P(s) = \exp\left[-\int_0^s \mu(\tau, \tau + C)\,\mathrm{d}\tau\right]P(0)$$

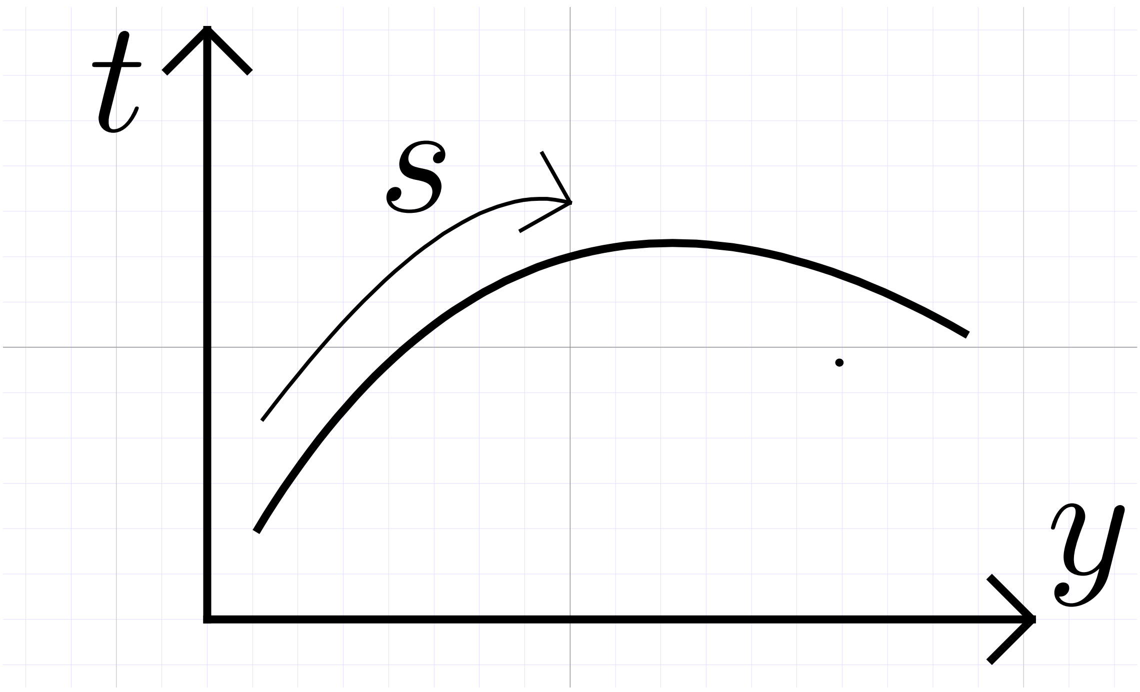

Figure 1.