Study of neuron models

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Abstract

This report describes the classic Hodgkin-Huxley model of neuron electrochemical activity that leads to synapses, extended to consideration of unidimensional propagation along a sequence of neurons.

1 Introduction

Multicellular biological organisms with differentiated cells need to establish information transfer between various specialized cells. We consider an extension of the classical Hodgkin-Huxley electrochemical neuron model [?] to series of neurons.

1.1 Neuron physiology

1.1.1 Neuron structure

Figure 1. Basic structure of a neuron.

As shown in Fig. 1, a neuron is a specialized cell with an elongated extension called an axon and several shorter extensions called dendrites.

1.1.2 Electrochemistry of the neuron

- specific membrane permeability for different ions (K^+, Na^+, Cl^-)
- neuron membranes exhibit polarization achieved by: passive ion transport and active ion transport mediated by Na/K ion channel pumps

- Neuron structure maintains a resting polarization voltage across an axon membrane of $V_r = -70$ mV.

1.2 Hodgkin-Huxley model

1.2.1 Equivalent electrical circuit

Figure 2.

1.2.2 Ion transport across neuron membrane

1.3 Propagation of electrical impulses along neuron axon

2 Methods

2.1 Change of ion concentration in axon cross-section

Assuming constant ion concentrations in an axon cross-section the rate of change of K^+, Na^+, Cl^- concentrations is given by the ODE system

- $$\frac{\mathrm{d}m}{\mathrm{d}t} = \alpha_m(V)(1-m) - \beta_m(V)m$$
- $$\frac{\mathrm{d}n}{\mathrm{d}t} = \alpha_n(V)(1-n) - \beta_n(V)n$$
- $$\frac{\mathrm{d}h}{\mathrm{d}t} = \alpha_h(V)(1-h) - \beta_h(V)h$$

The equations express a two time scale concentration evolution between values of 0 and 1 with time scales given by α_i, β_i respectively (Fig.)

2.2 Membrane voltage

2.3 Driving force

Ions are transported across the axon membrane due to a voltage difference. Transport occurs across ion channels, both passively and actively. The overall rate of change of voltage is given by

- $$C_m \frac{\mathrm{d}V}{\mathrm{d}t} = \bar{g}_{\mathrm{Na}} m^3 h\,(V - E_{\mathrm{Na}}) + \bar{g}_{\mathrm{K}} n^4\,(V - E_{\mathrm{K}}) + g_\ell\,(V - E_\ell) + P(t)$$

```
In[20]:= iNa = gNA (m[t])^3 h[t] (V[t] - ENa);
         iK = gK (n[t])^4 (V[t] - EK);
         iCl = gl (V[t] - El);
```

```
In[23]:= ODE4 = Cm V'[t] == iNa + iK + iCL + P[t];
```

```
In[24]:=
```

- The resting voltages for the individual ions are $E_{\mathrm{Na}}=55$ mV, $E_K=-82$ mV, $E_\ell=-59$ mV
- Ion conductance constants are $\bar{g}_{\mathrm{Na}}=70.7$ (m-mhos/cm^2), $\bar{g}_K=24.34$ (m-mhos/cm^2), $g_\ell=0.3$ (m-mhos/cm^2).
- The membrane capacitance is $C_m=0.001$ (F/cm^2)

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In[8]:=
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In[9]:=
In[20]:=
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In[11]:=
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In[12]:=
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In[13]:=
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In[14]:=
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In[15]:= ODEsys = {ODE1,ODE2,ODE3,ODE4};
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In[16]:= IC = {m[0]==1,n[0]==0.5,h[0]==0.25,V[0]=-70};
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```
In[21]:= am[V_]=V; an[V_]=V; al[V_]=V; bm[V_]=1; bn[V_]=1; bl[V_]=V;
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In[18]:= gNA=70.7; gK=24.34; gl=0.5;
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In[19]:= P[t_]=0;
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In[22]:= sol=NDSolve[Flatten[ODEsys,IC],{m[t],n[t],l[t],V[t]},{t,0,1}];
```

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 Flatten::flpi: Levels to be flattened together in m[0] == 1, n[0] ==
0.5, h[0] == 0.25, -70 should be lists of positive integers.
NDSolve::deqn: Equation or list of equations expected instead of
Flatten[m'[t] == -m[t] + (1 - m[t]) V[t], n'[t] == -n[t] + (1 - n[t])
V[t], h'[t] == ah[<<1>>] <<1>> + <<1>>, 4 3 Cm V'[t] == iCL + n[t] (-EK +
V[t]) + h[t] m[t] (-ENa + V[t]), <<4>>] in the first argument
Flatten[m'[t] == -m[t] + (1 - m[t]) V[t], n'[t] == -n[t] + (1 - n[t])
V[t], h'[t] == ah[<<1>>] <<1>> + <<1>>, 4 3 Cm V'[t] == iCL + n[t] (-EK +
V[t]) + h[t] m[t] (-ENa + V[t]), <<4>>].
```

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In[23]:=
```

3 Results

4 Conclusions

Bibliography