Overview

Note: Presentation is simpler than that in textbook

- Interval subpartitions
- Piecewise Lagrange basis
- Analysis: rate of convergence

- Two levels of partitioning for $f\colon [a,b] \to \mathbb{R}$. The idea is to use low-degree polynomials over subintervals.
  - *Overall partition* $[a,b] = \bigcup_{j=1}^{n} [a_j, b_j]$, $a = a_1 < a_2 < \cdots < a_n = b - l$. Options:
    - → If function does not exhibit regions of large variability: *uniform partition*

$$a_j = a + (j-1)l, b_j = a_j + l, l = (b-a)/n$$

    - → If function varies rapidly in subregions: *adaptive partition*
  - *Interval subpartition*, almost always uniform, m chosen small, ($m \leqslant 5$)

$$[a_j, b_j] = \bigcup_{j=0}^{m} [a_{ij}, b_{ij}], a_{ij} = a_j + ih = b_{i-1,j}$$

Function sample points are chosen as:

- → For piecewise constant: $x_{0j} = (a_j + b_j)/2$ (midpoint)
- → For $m > 0$, $x_{ij} = a_j + ih_j$, $h_j = (b_j - a_j)/m$

- Idea: m sample points over each of n partition intervals:
  - Interpolation can be discontinous at subinterval edges $a_2, \ldots, a_n$
  - Discontinuity in function or derivatives is often required (corners, jumps)
- Over interval $[a_j, b_j]$, barycentric Lagrange interpolation is

$$p_j(t) = \frac{\sum_{i=0}^{m} y_{ij} \dfrac{w_{ij}}{t - x_{ij}}}{\sum_{i=0}^{m} \dfrac{w_{ij}}{t - x_{ij}}}, \, w_{ij} = \prod_{k=0, k \neq i}^{m} \frac{1}{x_{ij} - x_{kj}}$$

- Weights are easily precomputed for uniform subpartitions $x_{ij} = a_j + i\, h_j$

$$\frac{1}{w_{ij}} = \prod_{k=0, k \neq i}^{m} x_{ij} - x_{kj} = (h_j)^m \prod_{k=0}^{i-1} (i - k) \prod_{k=i+1}^{m} (i - k) \Rightarrow$$

$$\frac{1}{w_{ij}} = (-1)^{m-i} (h_j)^m \left(\prod_{s=0}^{i} s \right) \left(\prod_{s=1}^{m-i} s \right) = (-1)^{m-i} (h_j)^m \, i! \, (m - i)!$$

- On interval $[a_j, b_j]$ Lagrange basis is: $\{l_{0j}(t), \ldots, l_{mj}(t)\}$, $j = 1, \ldots, n$.
- Outside interval $[a_j, b_j]$ set $l_{ij}(t) = 0$, $i = 0, 1, \ldots, m$.
- Gather into single basis set

$$[\; l_{01}(t) \;\; l_{11}(t) \;\; \ldots \;\; l_{m1}(t) \;\; l_{02}(t) \;\; l_{12}(t) \;\; \ldots \;\; l_{m2}(t) \;\; l_{0n}(t) \;\; l_{1n}(t) \;\; \ldots \;\; l_{mn}(t) \;]$$

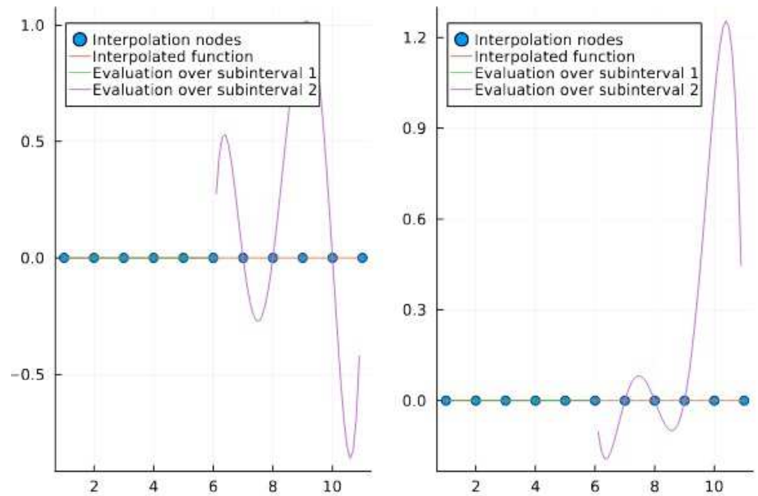

Figure 1. Two members of a piecewise Lagrange basis of degree 5 over two subintervals