MATH566
Introduction to Numerical Analysis

Course Syllabus

Times	MWF 2:30-3:20PM, Hanes 125
Office hours	MW 1:00-2:00PM, F 3:30-4:30PM, CP451
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Historical context and motivation

Mathematics is a branch of knowledge dealing with abstract concepts such as quantity, structure, change, or patterns. Mathematical objects are defined by specific properties attached to such concepts and operations are introduced to work with these objects. Statements about the objects can be either axioms assumed as true, or theorems derived through formal logic from object definitions and axioms.

Euclidean geometry furnishes a classical example within which objects such as points, lines, polygons, circles or cubes are defined. Purely geometric statements are made about these objects such as the axiom that a single line passes through two distinct points or the theorem that two lines in the plane are either parallel or intersect at a single point. Besides these geometric relationships, Euclidean geometry introduces notions of quantity through the length of a line segment or areas of plane figures. Identifying numbers as lengths of line segments is a remarkable extension of the notion of counting embodied in the natural numbers $\mathbb{N}^* = \{1, 2, 3, \dots\}$, and allows definition of the rational numbers $\mathbb{Q}$ as well as the irrationals $\mathbb{R} - \mathbb{Q}$. Within Euclidean geometry quantities are to be derived by operations with a ruler and a compass, since these conform to the allowed primitive objects.

As in all mathematical formalisms, many Euclidean geometry results can be established by *symbolic* operations with the defined objects. Such results are often called *analytic*, in that they are obtained by breaking apart a complex problem into smaller pieces. Solutions obtained in this manner are considered *exact*, in the sense that they conform to the accepted formal rules. However, not all results can be obtained by purely symbolic operations. In the context of Euclidean geometry the famous "unsolvable problems" include:

- the Delian problem of doubling the cube, find the edge length needed to double a cube's volume;
- squaring of the circle, find the square of the same area as a circle;
- trisecting an angle, breaking apart an angle into three equal arcs.

Though the above problems cannot be solved symbolically through ruler and compass operations, they all do have well defined solutions. These solutions are no longer exact in that they fall outside the accepted formal rules. They are therefore called *approximations*, and numerical approximations of quantities are the most commonly encountered. For example, the Delian problem is solved by constructing the sequence of ever more accurate approximations defined by $x_0 = 1$,

$$x_{n+1} = \frac{2}{3}\left(x_n + \frac{1}{x_n^2}\right), n = 0, 1, 2, \ldots$$

with $\lim_{n\to\infty} x_n = 2^{1/3}$.

It is the objective of *numerical analysis* to define approximation of quantities arising in various branches of mathematics, and to establish the properties of these approximations. With reference to the above sequence, such properties might include establishing convergence or error for given n. It should be noted that Gödel's Incompleteness Theorem states that all mathematical formalisms include results that cannot be established with allowed symbolic operations. Indeed, application of mathematics to practical problems almost always requires a numerical approach.

Course goals

Students will acquire proficiency in numerical approximation of problems arising from:

- univariate and multivariate calculus;
- linear algebra.

Specific skills that will be acquired include:

- identification of appropriate approximation method;
- analysis of convergence behavior;
- analysis of computational effort require to achieve desired precision;
- effects of inexact arithmetic;
- implementation of approximation method;
- use of numerical approximation software.

Honor Code

Unless explicitly stated otherwise, all work is individual. You may discuss various approaches to homework problems with students, instructors, but must draft your answers by yourself. The use of automated tools to construct solutions such as code generation by generative AI (e.g., Chat GPT) with appropriate attribution and referencing is encouraged and fully incorporated in the course presentation, assignments, and examinations.

Grading

Required work

- Homework - Best 10 assignments x 5 = 50 points
- Midterm examination = 10 points
- Final examination = 40 points
- Absence accomodation - additional two homework assignments, 2 x 5 = 10 points

Mapping of point scores to letter grades

Grade	Points	Grade	Points	Grade	Points	Grade	Points
H+,A *cum laude*	101-110	H-,B+	86-90	P-,C+	71-75	L-,D+	56-60
H+,A	96-100	P+,B	81-85	L+,C	66-70	L--,D-	50-55
H,A-	91-95	P,B-	76-80	L,C-	61-65	F	0-49

Extra credit

- Various in-class announcements as well as further study topics posted in this space. The total number of additional homework assignment or extra credit points is capped at 10 points.
- EC1 (10 pts): Read the paper *The Chebyshev points of the first kind* by Kuan Xu and draft a summary. Implement first and second type Chebyshev approximants and apply to the solution of either integral or differential equations. It is recommended to use Julia code embedded in a TeXmacs document. The overall document should be approximately 4 pages in length, and is a an excellent inital foray into numerical analysis that can be further pursued as a Senior Honor Thesis and introduction to computational science research.

Examinations

- A midterm examination during normal class meeting time on Wednesday, October 16. (MidtermSolution)
- The final examination, Friday, December 6, 8:00AM. (FinalSolution)

Course policies

- Class attendance is expected and highly beneficial to understanding of course topics. There is no need to inform instructor of planned absences.

- Course grade is based upon accumulation of credit points (0-100). There is no "grading on a curve". Extra credit opportunities are offered for an additional 10 grade points, to allow for missed homework or tests.
- Homework is to be submitted electronically through Canvas. E-mailed homework is not accepted. Late homework is accepted only in the case of University approved class absences. Two supplementary homework assignments should provide sufficient flexibility for individual circumstances. E-mail messages requesting acceptance of late homework due to any other circumstance are deleted without review or response. Students are advised to prepare and submit homework well in advance of the Canvas deadline to allow for unforseen difficulties. Suspension of classes due to campus-wide events (weather, pandemic, etc.) will lead to modification of due dates or elimination of specific assignments for the entire class.

Accessibility resources and services. The University of North Carolina at Chapel Hill facilitates the implementation of reasonable accommodations, including resources and services, for students with disabilities, chronic medical conditions, a temporary disability or pregnancy complications resulting in barriers to fully accessing University courses, programs and activities.

Accommodations are determined through the Office of Accessibility Resources and Service (ARS) for individuals with documented qualifying disabilities in accordance with applicable state and federal laws. See the ARS Website for contact information: https://ars.unc.edu or email ars@unc.edu.

Counseling and psychological services (CAPS). CAPS is strongly committed to addressing the mental health needs of a diverse student body through timely access to consultation and connection to clinically appropriate services, whether for short or long-term needs. Go to their website: https://caps.unc.edu/ or visit their facilities on the third floor of the Campus Health Services building for a walk-in evaluation to learn more.

Title IX resources. Any student who is impacted by discrimination, harassment, interpersonal (relationship) violence, sexual violence, sexual exploitation, or stalking is encouraged to seek resources on campus or in the community. Reports can be made online to the EOC at https://eoc.unc.edu/report-an-incident/. Please contact the University's Title IX Coordinator (Elizabeth Hall, interim – titleixcoordinator@unc.edu), Report and Response Coordinators in the Equal Opportunity and Compliance Office (reportandresponse@unc.edu), Counseling and Psychological Services (confidential), or the Gender Violence Services Coordinators (gvsc@unc.edu; confidential) to discuss your specific needs. Additional resources are available at safe.unc.edu.

Generative AI. Use of automated tools to construct solutions and coding assigments is encouraged. All such use must be clearly identified and attributed to the original source. Examples will be provided of proper attribution and identification of sources. Grading is based upon analysis of results obtained from such tools.

Course materials

Course topics

- Floating point arithmetic (FPA)

- Iteration (ITR)
- Julia software (JUL)
- Polynomial interpolation (IPL)
- Roots of univariate nonlinear functions (ZER)
- Spline interpolation (SPL)
- Least squares approximation (LSQ)
- Chebyshev approximation (CHB)
- Nonlinear systems of equations (SYS)
- Numerical differentiation (DIF)
- Numerical integration (INT)
- Numerical solution of differential initial value problems (ODE)
- Numerical solution of differential boundary value problems (BVP)
- Numerical solution of partial differential equations (PDE)
- Course review (REV)

Textbook

All course concepts are presented on slides below. The following textbook is freely available in electronic form from the UNC library.

- *Numerical analysis: theory and experiments*, David Sutton

Class slides

Slides summarizing the main topics of each lecture are generally posted 48 hours prior to class time starting in Week 2. It is useful to glance at these before class to better assimilate the material, ask clarifying questions during class time. Relevant sections from *Numerical analysis: theory and experiments* are indicated similarly to §1.2.

Week	Date	Topic		
01	08/19	FPA & JUL	Lesson01 §1, §2 L01.jl L01	Lesson02 §5 L02.jl L02
02	08/26	ITR & JUL	Lesson03 §2, 3 L03.jl L03	Lesson04 §3, 4 L04.jl L04
03	09/04	IPL	Lesson05 §6.1-2 L05.jl L05	Lesson06 §6.3-5 L06.jl L06
04	09/09	SPL	Lesson07 notes L07.jl L07	Lesson08 §7,8 L08.jl L08
05	09/16	CHB	Lesson09 §10 L09.jl L09	Lesson10 §11 L10.jl L10
06	09/23	LSQ	Lesson11 L11.jl L11	Lesson12
07	09/30	ZER	Lesson13 L13.jl L13	Lesson14 §31 L14.jl L14
08	10/07	SYS	Lesson15 L15.jl L15	Lesson16 L16.jl L16
09	10/14	REV	Midterm review Practice Midterm Solution	Midterm examination
10	10/21	DIF	Lesson17 L17.jl L17	Lesson18 §13 L18.jl L18
11	10/28	INT	Lesson19 §14 L19.jl L19	Lesson20 §15 L20.jl L20
12	11/04	INT	Lesson21 L21.jl L21	Lesson22 L22.jl L22
13	11/11	ODE	Lesson23 §22,23 L23.jl L23	Lesson24 §24 L24.jl L24 BL.nb
14	11/18	BVP	Lesson25 L25.jl L25	Lesson26 L26.jl L26
15	11/25	PDE	Lesson27 L27.jl L27	Lesson28 L28.jl L28
16	12/02	REV		

Homework

Homework is assigned through Canvas. Homework is submitted as Pluto notebooks.

Nr.	Issue Date	Problems	Due Date	Solution	Topic
01	08/23	H01.jl	08/30	S01	FPA
02	09/03	H02.jl	09/06	S02	ITR
03	09/06	H03.jl	09/13	S03	IPL
04	09/18	H04.jl	09/30	S04	SPL
05	10/2	H05.jl	10/9	S05	CHB
06	09/27	H06.jl	10/04	S06	MID
07	10/25	H07.jl	11/01	S07	DIF
08	11/01	H08.jl	11/08	S08	INT
09	11/08	H09.jl	11/15	S09, H09.nb	INT
10	11/19	H10.jl	11/22	S10	ODE
11	11/22	H11.jl	11/26	S11	ODE
12	11/25	H12.jl	12/04	S12	BVP

Software

The course will use the Julia language within interactive Pluto notebooks to implement and study numerical methods. Follow the website instructions to install Julia and Pluto on your CCI-compliant laptop.