

LINEAR ALGEBRA TOOLS - EXERCISES

1. Linear mappings

Exercise 1. Construct the matrix A that represents the mapping $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, f reflects a vector across the x_1x_2 plane.

Exercise 2. Construct the matrix B that represents the mapping $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, f reflects a vector across the x_2x_3 plane.

Exercise 3. Construct the matrix C that represents the mapping $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, f reflects a vector across x_2 axis.

Exercise 4. Does $C = BA$? If so, what does this signify?

Exercise 5. Construct the matrix D that represents the mapping $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, f rotates a vector around the x_3 axis by angle θ .

Exercise 6. Construct the matrix E that represents the mapping $f: \mathbb{R}^3 \rightarrow \mathbb{R}^3$, f rotates a vector around the x_2 axis by angle φ .

Exercise 7. What does DE represent?

Exercise 8. What does ED represent?

Exercise 9. Is $DE = ED$ true? Explain

2. Norms

Exercise 10. For $x \in \mathbb{R}^m$, prove $\|x\|_\infty \leq \|x\|_2$.

Exercise 11. For $x \in \mathbb{R}^m$, prove $\|x\|_2 \leq \sqrt{m} \|x\|_\infty$.

Exercise 12. Prove that equality holds in $\|x + y\|_2 \leq \|x\|_2 + \|y\|_2$ if and only if $\exists a < \mathbb{R}$, s.t. $x = ay$.