MATH661 HW00 - Number approximation

Posted: 08/30/23
Due: 09/06/23, 11:59PM
Tracks 1 & 2: 1-4. **Track 2**: 5.

This homework introduces the fundamentals of additive approximation techniques in vector spaces. Read and understand the concepts in L04: Linear combinations in $\mathcal{R}_m$ *and* $\mathcal{C}^0[0,2\pi)$*, in particular to the example presented in the text and in the code attached to L04: Figure 2. Additional Julia coding constructs are also introduced. Remember to always execute the code snippets within the lecture notes to understand Julia programming techniques.*

1. Approximate $b(t)=t(\pi-t)(2\pi-t)$ on the interval $[0,2\pi)$ by a cosine series

$$b(t)\cong\sum_{k=1}^{n} y_k\cos(kt), A(t)=[\ 1\ \ \cos(t)\ \ \cos(2t)\ \ \ldots\ \ \cos(nt)\ ], A:\mathbb{R}\to\mathbb{R}^{n+1}.$$

   Study the convergence behavior of the approximation.

   Solution.

2. Approximate $b(t)=t(\pi-t)(2\pi-t)$ on the interval $[0,2\pi)$ by a trigonometric series

$$b(t)\cong\sum_{k=1}^{n} [x_k\cos(kt)+y_k\sin(kt)], A(t)=[\ 1\ \ \cos(t)\ \ \sin(t)\ \ \cos(2t)\ \ \sin(2t)\ \ \ldots\ \ \cos(nt)\ \ \sin(nt)\ ],$$

   $A(t):\mathbb{R}\to\mathbb{R}^{2n+1}$. Study the convergence behavior of the approximation.

   Solution.

3. Approximate $b(t)=t\cdot|\pi-t|\cdot|2\pi-t|$ on the interval $[0,2\pi)$ by a sine series

$$b(t)\cong\sum_{k=1}^{n} y_k\sin(kt), A(t)=[\ 1\ \ \cos(t)\ \ \cos(2t)\ \ \ldots\ \ \cos(nt)\ ], A:\mathbb{R}\to\mathbb{R}^{n+1}.$$

   Study the convergence behavior of the approximation.

   Solution.

4. Approximate $b(t)=t\cdot|\pi-t|\cdot|2\pi-t|$ on the interval $[0,2\pi)$ by a sawtooth series

$$b(t)\cong\sum_{k=1}^{n} z_k\sin(kt), A(t)=[\ 1\ \ \cos(t)\ \ \cos(2t)\ \ \ldots\ \ \cos(nt)\ ], A:\mathbb{R}\to\mathbb{R}^{n+1}.$$

   Study the convergence behavior of the approximation.

   Solution.

5. Completely state the mathematical problem of solving the initial value problem for an ordinary differential equation of first order. Use Lyapunov exponents [1] to find the absolute condition number.

   Solution.

Bibliography

[1] Luís Barreira. *Lyapunov Exponents*. Springer International Publishing, Cham, 2017.
[2] Girolamo Cardano. *Ars magna or The rules of algebra*. New York : Dover, 1993.