MATH661 HW06 - Interpolation

Posted: 10/11/23
Due: 10/18/23, 11:59PM

Beginning with this assignment, homework tasks are described at a higher level. Apply your experience from previous assignments to cogently formulate a solution.

1 Track 1 & 2

Study the convergence of polynomial interpolation, $p_n(x_i) = f(x_i)$, $i = 1, 2, \ldots, n$ with increasing number of sample points n for the following functions:

1. $f\colon [-1,1] \to \mathbb{R},\ f(t) = \cos(\pi t/2)$;
2. $f\colon [-1,1] \to \mathbb{R},\ f(t) = 1/(1+25t^2)$;
3. $f\colon [-1,1] \to \mathbb{R},\ f(t) = \exp(-25t^2)$.

For each case consider both equidistant and Chebyshev sample points, present plots of the function and interpolant, plots of the error as a function of n, and compare the observed error with that predicted by the formula

$$e(t) = f(t) - p(t) = \frac{f^{(n+1)}(\xi_t)}{(n+1)!} \prod_{i=0}^{n} (t - x_i).$$

Comment on what you observe.

2 Track 1

For $n = 5$ equidistant sample points, explicitly write the Lagrange and Newton forms of the interpolating polynomial.

3 Track 2

Repeat the above convergence study for Hermite interpolation $p_n(x_i) = f(x_i)$, $p_n'(x_i) = f'(x_i)$, $i = 1, 2, \ldots, n$.