MATH661 HW09 - Multiple operator approximation

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Investigate various aspects of the $Ly = f(y)$ problem with L a linear operator and f nonlinear. Algorithms will be applied to the Van der Pol oscillator for $t \in [0, 250]$ with $\mu = 1$,

$$x'' - \mu(1 - x^2)x' + x = 0, x(0) = 0, x'(0) = 1.$$

Plots of the solution are typically presented in phase space as (x(t),x'(t)).

1 Track 1

1. Rewrite the second-order ODE as a system of first-order equations $\boldsymbol{y}' = \boldsymbol{f}(t, \boldsymbol{y})$.

2. Carry out a convergence study when applying the single-step Runge-Kutta method

$$\boldsymbol{y}_{n+1} = \boldsymbol{y}_n + \frac{1}{6}(\boldsymbol{k}_1 + 2\boldsymbol{k}_2 + 2\boldsymbol{k}_3 + \boldsymbol{k}_4),$$

$$\begin{aligned}
\boldsymbol{k}_1 &= h\boldsymbol{f}(t_n, \boldsymbol{y}_n) \\
\boldsymbol{k}_2 &= h\boldsymbol{f}\left(t_n + \frac{1}{2}h, \boldsymbol{y}_n + \frac{1}{2}\boldsymbol{k}_1\right) \\
\boldsymbol{k}_3 &= h\boldsymbol{f}\left(t_n + \frac{1}{2}h, \boldsymbol{y}_n + \frac{1}{2}\boldsymbol{k}_2\right) \\
\boldsymbol{k}_4 &= h\boldsymbol{f}(t_n + h, \boldsymbol{y}_n + \boldsymbol{k}_3)
\end{aligned}$$

3. Carry out a convergence study when applying the Adams-Bashforth of order 4.

2 Track 2

1. Rewrite the second-order ODE as a system of first-order equations $\boldsymbol{y}' = \boldsymbol{f}(t, \boldsymbol{y})$.

2. Use a symbolic package (e.g., Mathematica) to verify the fourth-order theoretical accuracy of the Runge-Kutta method from Track 1.

3. Carry out a convergence study using Adams-Moulton of fourth order. This requires knowledge the latest value $\boldsymbol{y}_{n+1}$. Approximate this by a predictor $\tilde{\boldsymbol{y}}_{n+1}$ obtained by an Adams-Bashforth of fourth order where needed, e.g., $\boldsymbol{f}(\boldsymbol{y}_{n+1}) \cong \boldsymbol{f}(\tilde{\boldsymbol{y}}_{n+1})$.