MATH661 HW10 - Nonlinear operators

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The simplest nonlinear operator is a scalar function $f(x)$, and a basic problem is to find the null set of f, those values x for which $f(x)=0$, known as the roots of f.

Consider $f(x)=p_8(x)=\sum_{i=0}^{8} a_i x^i$, an eigth degree polynomial with $\boldsymbol{a}=[\ a_0\ \ \ldots\ \ a_8\ ]$

$$\boldsymbol{a}=[\ 40320\ \ -109584\ \ 118124\ \ -67284\ \ 22449\ \ -4536\ \ 546\ \ -36\ \ 1\ ].$$

1 Track 1

Implement each of the following methods to find a root of p_8.

1. Seek a root $r\in[5.5,6.5]$ using the bisection algorithm (see course webpage).
2. Seek r by the secant method.
3. Seek r by Newton's method.
4. Seek r by Steffensen's method.
5. Change $a_7=-36-10^{-3}$ and repeat the above

2 Track 2

1. Prove that Steffensen's method is of second order.
2. Implement Steffensen's and find $r\in[5.5,6.5]$, a root of a perturbed $\tilde{p}_8(t)$, where $\tilde{a}_2=a_2+\varepsilon_k$, $\varepsilon_k=2^{-k}$, $k\in\{15,14,\ldots,10\}$. Comment on what you observe.
3. Apply the vector-valued version of Newton's method

$$\boldsymbol{x}_{n+1}=\boldsymbol{x}_n-\boldsymbol{J}_n^{-1}\,\boldsymbol{f}(\boldsymbol{x}_n), \tag{1}$$

where $\boldsymbol{J}_n$ is the Jacobian

$$\boldsymbol{J}_n=\boldsymbol{f}'(\boldsymbol{x}_n)=\frac{\partial\boldsymbol{f}}{\partial\boldsymbol{x}}(\boldsymbol{x}_n),$$

to find a root of

$$\begin{cases} uv-w^2=1 \\ uvw-u^2+v^2=2 \\ e^u-e^v+w=3 \end{cases}.$$

Implementation notes:

- As usual, $\boldsymbol{J}^{-1}$ is implemented as linear system solve

$$\boldsymbol{J}\boldsymbol{s}=\boldsymbol{f}(\boldsymbol{x}_n)\Rightarrow\boldsymbol{x}_{n+1}=\boldsymbol{x}_n+\boldsymbol{s}$$

- An initial guess is typically obtained by linearization of the system.