

MATH 661.FA23 MIDTERM EXAMINATION 2

Solve the problems for your appropriate course track. Problems probe understanding of the definitions and results from the module on function approximation through linear combination. Formulate your answers clearly, cogently, and include a concise description of your approach. Each question is meant to be completely answered within ten minutes. Allowed test time is 75 minutes.

1 Track 1

1. Construct the polynomial interpolant of data $\mathcal{D} = \{(-1, -6), (0, -1), (1, -2), (2, -3)\}$ in Lagrange form.
2. Construct the Newton form of the polynomial interpolant of the above data set, presenting the table of divided differences.
3. Efficiently evaluate the Newton form of the polynomial interpolant determined above at $t = 2$, using Horner's scheme. Present a pseudo-code algorithm.
4. Replace the sampling points $x_i = -1 + i$, $i = 0, \dots, 3$ in the data set \mathcal{D} so as to minimize the interpolation error over the interval $[1, 4]$.

2 Track 2

1. Construct the Hermite interpolant of data $\mathcal{D} = \{(x_i, y_i = f(x_i), y'_i = f'(x_i)), i = 0, 1\} = \{(-1, -6, 10), (0, -1, 1)\}$ in Newton form.
2. Construct the Hermite interpolant of the above data in the Lagrange form

$$p(t) = \sum_{i=0}^n [a_i(t) y_i + b_i(t) y'_i],$$

where $a_i(x_j) = \delta_{ij}$, $a'_i(x_j) = 0$, $b'_i(x_j) = \delta_{ij}$, $b_i(x_j) = 0$.

3. Present a spline interpolant S of data set $\mathcal{D} = \{(x_i = ih, y_i = f(x_i)), i = 0, \dots, n\}$, $h = 1/n$, where the restriction of S to interval $[x_{i-1}, x_i]$ is of the form

$$S_i(t) = a_i + b_i e^t + c_i e^{-t}.$$

4. Find the best inf-norm approximants of $f: [0, \pi/2] \rightarrow \mathbb{R}$, $f(t) = \cos t$ by polynomials of degree $n = 0, 1, 2$.