MATH 661.FA21 Midterm Examination 1

Solve the problems for your appropriate course track. Problems probe understanding of the definitions and results from the module on floating point arithmetic and linear algebra. Formulate your answers clearly, cogently, and include a concise description of your approach. Each question is meant to be completely answered within five minutes. Allowed test time is 75 minutes.

1 Common problems

1. Matrix $\boldsymbol{A} \in \mathbb{R}^{m \times n}$ has the singular value decomposition (SVD) $\boldsymbol{A} = \boldsymbol{U}\boldsymbol{\Sigma}\boldsymbol{V}^T$. Find the SVDs of:

   a) $(\boldsymbol{A}^T\boldsymbol{A})^{-1}$;

   b) $(\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T$;

   c) $\boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{A})^{-1}$;

   d) $\boldsymbol{A}(\boldsymbol{A}^T\boldsymbol{A})^{-1}\boldsymbol{A}^T$.

2 Track 1

1. Write pseudo-code to accurately evaluate the sum

$$S_{2n} = \sum_{k=1}^{2n} \frac{(-1)^{k+1}}{k} x^k$$

   in floating point arithmetic when $x = 1 + \varepsilon$, $1 \gg \varepsilon > 0$. ($\lim_{n\to\infty} S_{2n} = \ln(1+x)$).

2. Determine the operation count for the above algorithm.

3 Track 2

1. Let $\boldsymbol{A} \in \mathbb{R}^{m \times n}$. Show that the Moore-Penrose pseudoinverse $\boldsymbol{X} = \boldsymbol{A}^+$ minimizes $\|\boldsymbol{A}\boldsymbol{X} - \boldsymbol{I}\|_F$ over all n by m matrices.

2. Let $\boldsymbol{A} \in \mathbb{C}^{m \times m}$ be skew-Hermitian, i.e., $\boldsymbol{A}^* = -\boldsymbol{A}$. Prove that:

   a) $\boldsymbol{I} - \boldsymbol{A}$ is nonsingular;

   b) $\boldsymbol{C} = (\boldsymbol{I} - \boldsymbol{A})^{-1}(\boldsymbol{I} + \boldsymbol{A})$ is unitary.