

MATH661 - BACKGROUND QUIZ

Name:

1. Write the row echelon form \mathbf{E} of \mathbf{A} by observation, without carrying out any calculations

\mathbf{A}	\mathbf{E}
$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}$	$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$
$\begin{bmatrix} 2 & 0 & 1 \\ 1 & 0 & \frac{1}{2} \\ 0 & 4 & 0 \end{bmatrix}$	$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$
$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 3 & 0 & 0 \end{bmatrix}$	$\begin{bmatrix} & & \\ & & \\ & & \end{bmatrix}$

2. For $\mathbf{u} = [2 \ 1]^T$, $\mathbf{v} = [-1 \ 3]^T$, compute the following

a) $\mathbf{u}^T \mathbf{v}$

b) $\mathbf{u} \mathbf{v}^T$

3. For $f: [-1, 1] \rightarrow \mathbb{R}$, f differentiable an infinite number of times

a) Write the MacLaurin series of f (Taylor series around $x_0 = 0$)

b) What is the codomain of f ?

c) What is the domain of f ?

4. For $f: [-\pi, \pi] \times [-\pi, \pi] \rightarrow \mathbb{R}$, $f(x, y) = \sin x \cos y$ differentiable an infinite number of times

a) Write the MacLaurin series of f (Taylor series around $x_0 = 0$)

b) What is the gradient of f ?

5. Compute the products showing your calculation procedure

$$\mathbf{C} = \mathbf{A}\mathbf{B} = \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 2 \\ -1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -2 & 1 \\ 0 & 1 & 1 & 2 \\ -1 & 2 & 1 & 0 \\ 2 & 1 & 0 & 1 \end{bmatrix}, \mathbf{D} = \mathbf{B}\mathbf{A}.$$

6. Write pseudo-code that evaluates a polynomial through Horner's scheme

$$p_n(x) = (\dots ((a_n x + a_{n-1})x + a_{n-2})x + \dots + a_1)x + a_0$$

7. Write code in some language (e.g., Python, C, ...) to implement Horner's scheme.

8. After class, use ChatGPT to obtain an implementation of Horner's scheme.