Quiz 2

1. True or false? $f(x) = a\,x + b$, $f: \mathbb{R} \to \mathbb{R}$ is a linear mapping. Why?

   Answer. True if and only if $b = 0$. Verify linearity

$$\forall x, y, u, v \in \mathbb{R}: f(u x + v y) = u f(x) + v f(y) \Rightarrow$$

$$a(u x + v y) + b = u(a x + b) + v(a y + b) \Rightarrow b = 0$$

2. True or False? The 3x2 real-component matrix $\boldsymbol{A}$ can have maximal rank $r = 2$.

   Answer. True. Since $r = \dim C(\boldsymbol{A})$ and $\boldsymbol{A} \in \mathbb{R}^{3 \times 2}$ can have at most two linearly independent column vectors, e.g.,

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}.$$

3. True or false? $\boldsymbol{x} = [\,1 \;\; 1 \;\; 1\,]$ is an element of $C(\boldsymbol{A})$,

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

   Explain.

   Answer(s). False. Applying convention of column organization of vector components,

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \notin C\left(\begin{bmatrix} 1 & 0 & 1 \\ 1 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right).$$

   Imposing $\boldsymbol{x} = \boldsymbol{A}\boldsymbol{y}$ leads to third-component equation

$$1 = 0 \cdot y_1 + 0 \cdot y_2 + 0 \cdot y_3,$$

   contradiction.

   False. $\boldsymbol{x}$ is shown as a row vector.

4. True or false? It is known that $\boldsymbol{A}(\boldsymbol{x} + \boldsymbol{y}) = \boldsymbol{b}$ and $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$. Then $N(\boldsymbol{A}) = \{\boldsymbol{0}\}$. Explain. Answer. Ambigous. Subtracting equations gives $\boldsymbol{A}\boldsymbol{y} = \boldsymbol{0}$, hence $\boldsymbol{y} \in N(\boldsymbol{A})$. If $\boldsymbol{y} = \boldsymbol{0}$ then $N(\boldsymbol{A}) = \{\boldsymbol{0}\}$. If $\boldsymbol{y} \neq \boldsymbol{0}$ then $N(\boldsymbol{A}) \neq \{\boldsymbol{0}\}$.

5. True or False? $\boldsymbol{x} \in N(\boldsymbol{A})$ for

$$\boldsymbol{x} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \boldsymbol{A} = \begin{bmatrix} 0 & -1 & 2 \\ 0 & 1 & -2 \\ 1 & 0 & 2 \end{bmatrix}.$$

Answer. False. Compute

$$\boldsymbol{A}\boldsymbol{x} = 1 \cdot \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + 1 \cdot \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} \neq \mathbf{0}.$$