MATH661 Scientific Computation I

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Lesson 01

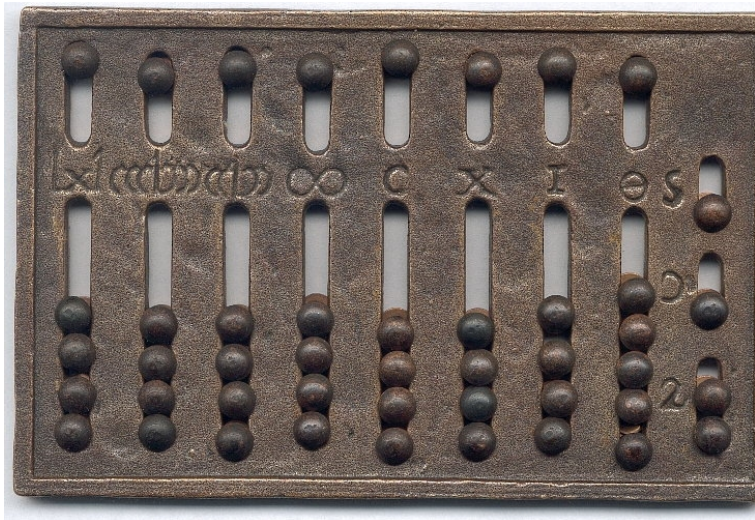
MATH661-L01

1 Scientific computation

MATH661-L01

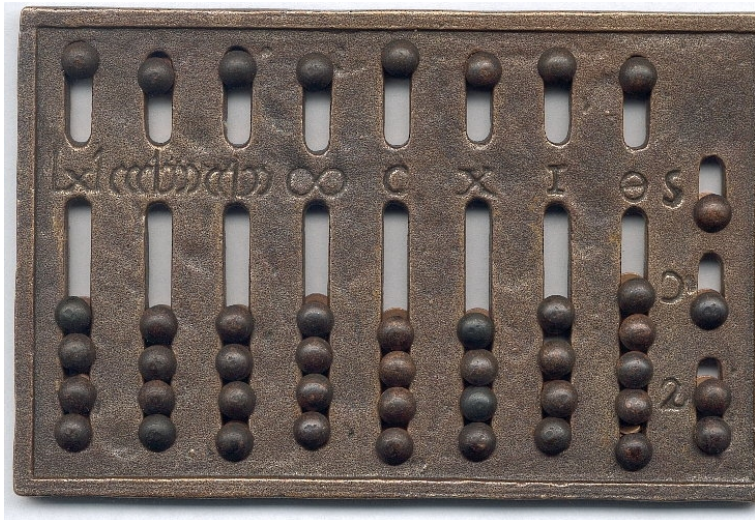
1. Scientific computation
   - History
   - Motivation
   - Approach

Applied mathematics



- Typical approach
  - Choose/develop appropriate mathematical theory for an application
  - Formulate a solution strategy
  - Find solutions under variety of assumptions
- Applied mathematics approaches
  - Analytical estimates
  - Numerical computation
  - Symbolic computation
  - Analog computation
- Computation devices
  - Mechanical devices from ca. 2500 BCE
  - Slide rule ca. 1620
  - AC/DC analog computers ca. 1900
  - Digital computers ca. 1950

Applied mathematics



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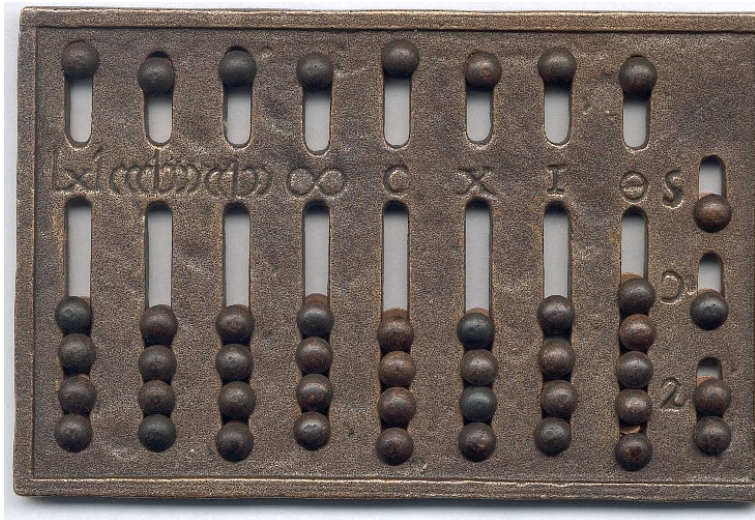

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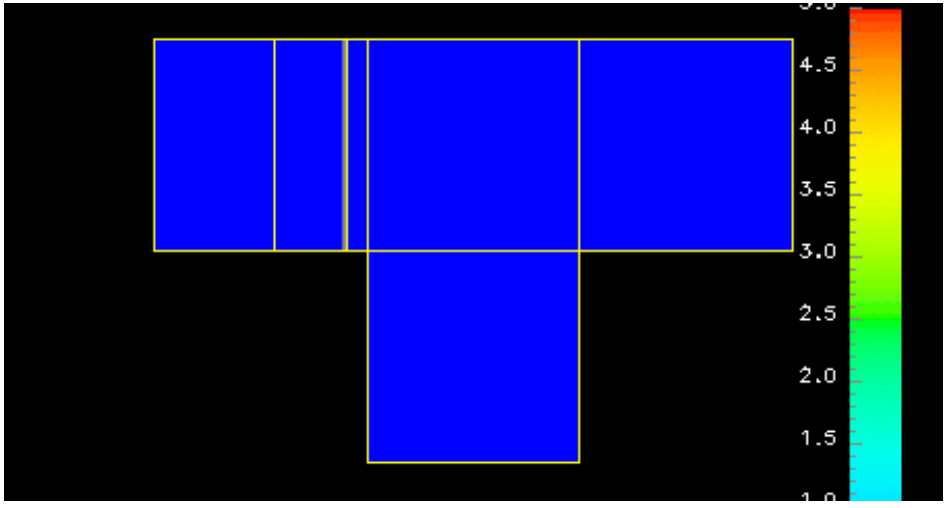
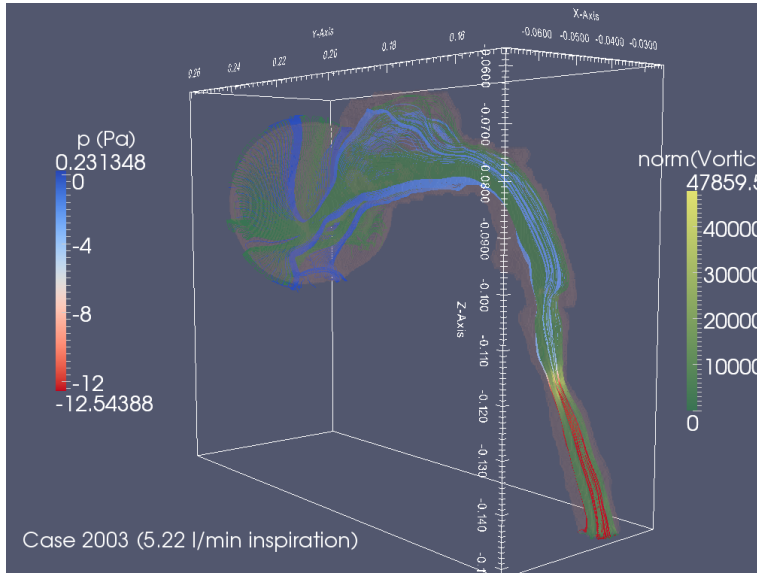
Model complicated phenomena

- Euler equations of gas dynamics
- Diffraction of shock wave over a cavity



Provide otherwise inaccessible data

- Medical data
- Subsurface models (e.g., natural gas reservoirs)
- Astrophysical computation



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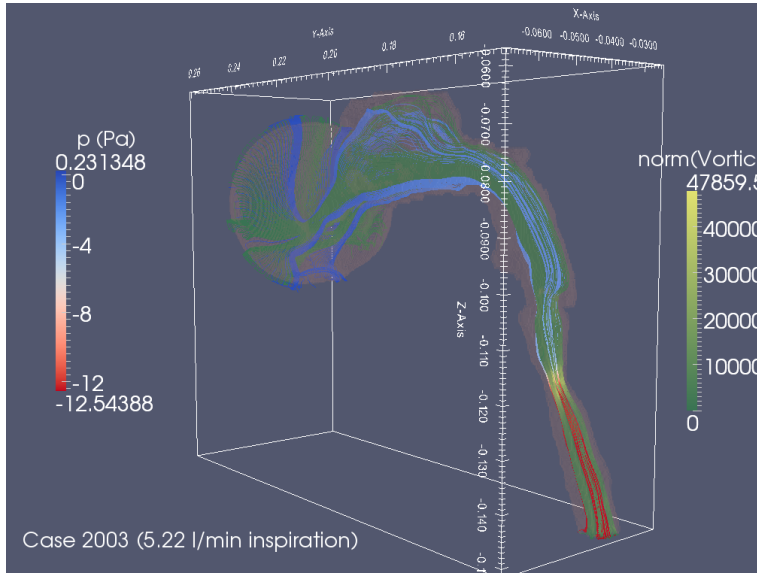

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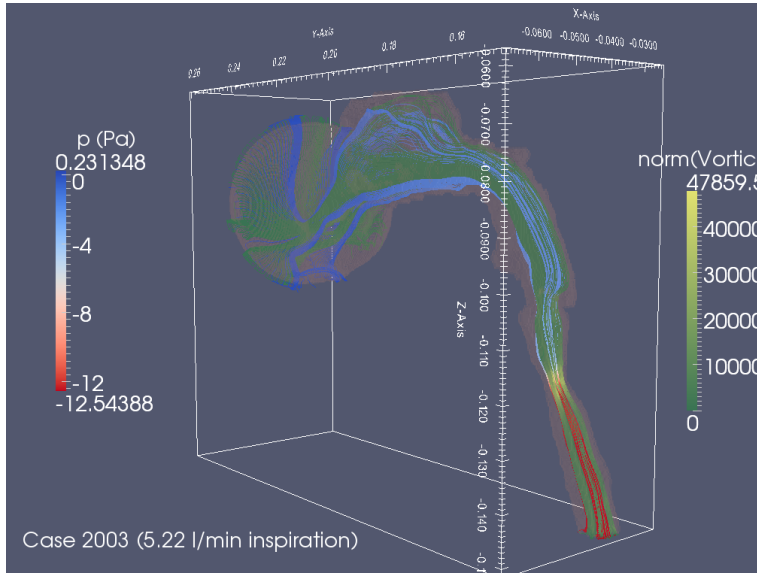
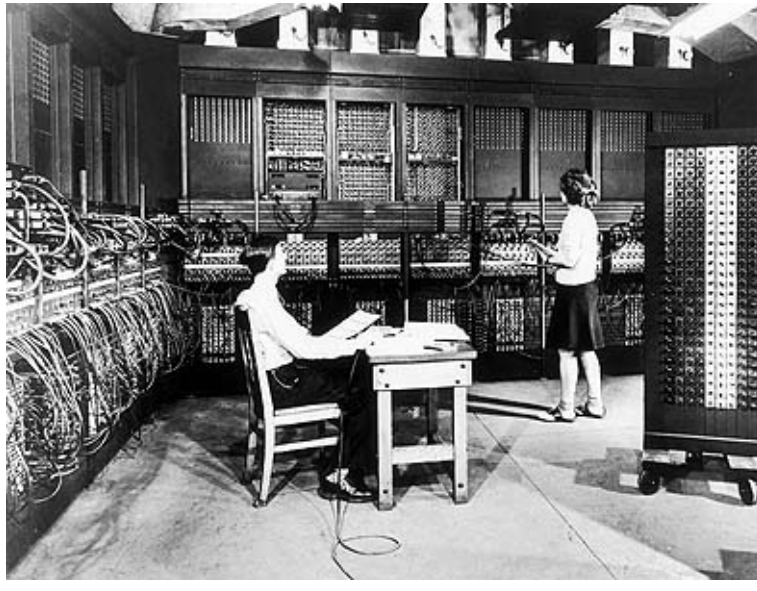


Digital numerical computation

- Approach
  - Digital computers are finite state machines
  - Introduce finite number systems
    - $\mathbb{I} \subset \mathbb{N}$, e.g. $\mathbb{I}_{64}$ a.k.a. long int
    - $\mathbb{F} \subset \mathbb{Q} \subset \mathbb{R}$, e.g. $\mathbb{F}_{64}$ a.k.a. double
  - Discretize problem of interest in $\mathbb{I}, \mathbb{F}$
  - Solve discretized problem
  - Interpret results



Questions

- What is the effect of using approximations of $\mathbb{N}, \mathbb{R}$?
- How do we establish correctness?
  - convergence
  - stability to errors
- How do we devise algorithms?
- What theoretical constructs are needed?

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