MATH661 Midterm Examination

Fall 2014 Semester, October 14, 2014

Instructions. *Answer the following questions. Provide a motivation of your approach and the reasoning underlying successive steps in your solution. Write neatly and avoid erasures. Use scratch paper to sketch out your answer for yourself, and then transcribe your solution to the examination you turn in for grading. Illegible answers are not awarded any credit. Presentation of calculations without mention of the motivation and reasoning are not awarded any credit. Each complete, correct solution to an examination question is awarded 4 course grade points. Your primary goal should be to demonstrate understanding of course topics and skill in precise mathematical formulation and solution procedures.*

1. Construct the spline second-degree polynomial interpolation of data $\mathcal{D} = \{(x_i = i\,h, y_i = f(x_i)), i = 0, \ldots, n\}$, $h = 1/n$, where $f\colon \mathbb{R} \to \mathbb{R}$, $f \in C^{(2)}(\mathbb{R})$ is a periodic function, $\forall x \in \mathbb{R}$ $f(x) = f(x+1)$. Estimate the number of floating point operations required to compute the spline interpolant.

Solution.

2. Construct the polynomial interpolant of data $\mathcal{D} = \{(-h,\ y_0 = f(-h)),\ (0,\ y_1 = f(0)), (0, y_1' = f'(0)), (h, y_2 = f(h))\}$, obtained by sampling a function $f\colon \mathbb{R} - \mathbb{R}$, $f \in C^{(3)}(\mathbb{R})$.

3. Determine the least squares approximation of data $\mathcal{D} = \{(x_i = i\,h, y_i = f(x_i)), i = 0, 1, \ldots, m\}$, $h = 2\pi/m$, $m \gg 2$, of form

$$p(t) = a_0 + a_1 \cos(t) + a_2 \sin(t),$$

with $f \in C^{\infty}(\mathbb{R})$, a periodic function, $\forall x \in \mathbb{R}$, $f(x) = f(x + 2\pi)$.