MATH661 Midterm Examination

Fall 2016 Semester, October 18, 2016

Instructions. *Answer the following questions. Provide a motivation of your approach and the reasoning underlying successive steps in your solution. Write neatly and avoid erasures. Use scratch paper to sketch out your answer for yourself, and then transcribe your solution to the examination you turn in for grading. Illegible answers are not awarded any credit. Presentation of calculations without mention of the motivation and reasoning are not awarded any credit. Each complete, correct solution to an examination question is awarded 4 course grade points. Your primary goal should be to demonstrate understanding of course topics and skill in precise mathematical formulation and solution procedures.*

1. (K&C 6.4.11, p. 362)

   i. Determine the values a, b, c such that

   $$f(x) = \begin{cases} 3 + x - 9x^2 & x \in [0,1] \\ a + b(x-1) + c(x-1)^2 + d(x-1)^3 & x \in [1,2] \end{cases},$$

   is a cubic spline with knots $t_0 = 0$, $t_1 = 1$, and $t_2 = 2$.

   ii. Determine d such $\| f'' \|_2$ is a minimum.

   iii. Find d from condition $f''(2) = 0$, and explain why it differs from value determined in (ii).

2. (K&C 7.1.14, p. 477) Determine the order of convergence with respect to h of the approximations

   i. $f'(x) \cong \frac{1}{2h}[-3f(x) + 4f(x+h) - f(x+2h)]$.

   ii. $f''(x) \cong \frac{1}{2h}[f(x+h) - 3f(x) + f(x-h)]$.

3. (K&C 7.3.12, p. 499) Consider the Gaussian quadrature approximation of integrals with symmetric bounds and even weight function $w(-x) = w(x)$,

   $$I(f) = \int_{-a}^{a} w(x) f(x) \mathrm{d}x \cong \sum_{i=0}^{n} A_i f(x_i).$$

   i. Prove that the quadrature nodes are symmetric with respect to the origin.

   ii. Prove that symmetric quadrature nodes have the same quadrature coefficients, $A_i = A_{n-i}$, $i = 0, 1, \ldots, n$.

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