MATH661 Homework 1 - Univariate approximation

Posted: Aug 23 Due: 11:55PM, Sep 6

1 Problem statement

Approximating a complicated function $f: \mathbb{R} \to \mathbb{R}$, by a simpler to compute function $g: \mathbb{R} \to \mathbb{R}$ is a fundamental operation within scientific computation. This homework investigates various techniques to solve this problem in the context of describing a 2D curve $(x(t), y(t))$.

Choose a contour of complexity comparable to the examples below that presents interest to you:

→   car silhouette (e.g., K&C, Fig. 6.6, p365)

→   blood vessel bifurcation

→   side profile of a rabbit or a whale.

Place your chosen contour in the figure space below, and display a table of points on the contour. Also record the value of derivatives (x_i', y_i') at some of the nodes.

Figure 1. Contour

i	x_i	y_i	x_i'	y_i'	i	x_i	y_i	x_i'	y_i'	i	x_i	y_i	x_i'	y_i'	i	x_i	y_i	x_i'	y_i'
0					5					10					15				
1					6					11					16				
2					7					12					17				
3					8					13					18				
4					9					14					19				

Table 1. Contour coordinates and tangent vectors

2 Theoretical exercises

1. K&C, 1.2.14, p.26 (include plots of the ODE solution for various c, and use Sage and Maxima to try to solve the ODE)

2. K&C, 1.2.25, p.26

3. K&C, 1.3.14-16, p.35

4. K&C, 6.1.31-32, p.326

5. K&C, 6.1.37, p327. Compare prediction to current prices. Present an analysis of any discrepancy

6. K&C, 6.3.13, p. 349. Repeat for $n = 3, 4$, using symbolic computation.

3 Implementation and analysis

1. Construct the Newton interpolation of the contour and compare to original.

2. Construct the Hermite interpolation of the contour and compare to original.

3. Construct the spline interpolation of the contour and compare to original.

4. Provide an analysis of the interpolation error and compare to your computational results.