By definition

$$\|\boldsymbol{A}\|_\infty = \sup_{\|\boldsymbol{x}\|_\infty = 1} \|\boldsymbol{A}\boldsymbol{x}\|_\infty,$$

and with above notation $\|\boldsymbol{x}\|_\infty = |x_k| = 1$. The vector $\boldsymbol{y} = \boldsymbol{A}\boldsymbol{x}$, $\boldsymbol{A} = [a_{ij}]$ has components

$$y_i = \sum_{j=1}^{n} a_{ij} x_i$$

and the inf-norm is

$$\|\boldsymbol{A}\boldsymbol{x}\|_\infty = \|\boldsymbol{y}\|_\infty = \max_i \left| \sum_{j=1}^{n} a_{ij} x_j \right|.$$

Consider the vector $\boldsymbol{x}$ with components $x_j = \operatorname{sign}(a_{ij})$ for which $\|\boldsymbol{x}\|_\infty = 1$. Then

$$\left| \sum_{j=1}^{n} a_{ij} x_j \right| = \sum_{j=1}^{n} a_{ij} x_j = \sum_{j=1}^{n} |a_{ij}|$$

and the maximum is obtained for the row vector of $\boldsymbol{A}$ of greatest 1-norm,

$$\|\boldsymbol{A}\|_\infty = \max_{1 \leqslant i \leqslant m} \|\boldsymbol{a}_i^*\|_1 .$$

Consider now the 2-norm definition

$$\|\boldsymbol{A}\|_2 = \sup_{\|\boldsymbol{x}\|_2 = 1} \|\boldsymbol{A}\boldsymbol{x}\|_2$$