MATH662
Numerical Linear Algebra

Course syllabus

Times	TuTh 11:00AM-12:15AM, Zoom synchronous meeting
Office hours	We 3:45PM-4:30PM, and by email appointment, Zoom
Instructor	Sorin Mitran

(*The instructor reserves the right to make changes to the syllabus. Any changes will be announced as early as possible.*)

Linear algebra is one of the fundamental techniques underlying multiple mathematical disciplines. Correspondingly, numerical linear algebra is the foundation of scientific computation. This course introduces the basic techniques, analysis methods, and implementation details of numerical linear algebra. The course emphasizes the strong link between theoretical concepts, algorithm formulation, and practical implementation that leads to the ubiquity of numerical linear algebra in applications. Recent progress in machine learning is based to a large degree on extending concepts from numerical linear algebra and will be discussed in the course.

Course goals

Upon course completion students:

- will be proficient in the basic operations of numerical linear algebra;
- will understand the significance of the main matrix factorizations;
- will be proficient in construction of linear subspaces;
- will be able to determine the computational complexity of numerical linear algebra algorithms;
- understand the links between numerical linear algebra operations and other fields of numerical analysis, in particular, approximation theory;
- will be able to implement numerical linear algebra algorithms.

Honor Code

Unless explicitly stated otherwise, all work is individual. You may discuss various approaches to homework problems with students, instructors, but must draft your answers by yourself.

Grading

Required work

- Bi-weekly homework: 6 assignments x 8 points = 48 points.
- In-class tests: 3 tests x 12 points = 36 points.
- Comprehensive final examination: 16 points.
- Extra credit: 4 reading topics x 3 points = 12 points.

Mapping of point scores to letter grades

Grade	Points	Grade	Points	Grade	Points	Grade	Points
H+,A+	101-112	H-,B+	86-90	P-,C+	71-75	L-,D+	56-60
H+,A,	96-100	P+,B	81-85	L+,C	66-70	L-,D-	50-55
H,A-	91-95	P,B–	76-80	L,C-	61-65	F	0-49

Course policies

- Class attendance is expected and highly beneficial to understanding of course topics.
- Homework is to be submitted electronically through Sakai.
- Late homework is not accepted.
- Students are offered the opportunity to make up for 12 course points (i.e., 3 homeworks, or 1 in-class test) through extra credits posted on this web page every 3 weeks. This should accomodate a reasonable number of excused absences.
- There is no need to inform instructor of planned absences.

Extra credit topics

For each of the topics below:

- read the textbook presentation
- look up and read original sources, use Web of Science
- try a small sample computation
- present influence of work in the field by following citations

1. Lecture 30, Jacobi
2. Lecture 30, Bisection
3. Lecture 30, Divide & conquer

Extra credit: 4 reading topics x 3 points = 12 points.

Examinations

- Three tests are scheduled during class hours, approximately once every 4 weeks, covering the material presented during that time period.
- The final examination covers *all* course material, and concentrates on verification of understanding of basic concepts rather than extensive computation or detalied knowledge of analytical techniques.
- Test1 and Test2 are closed-book. Test3 and the Final Examination are open-book. Test 3 is an implementation problem.
- Final examination: May 7, 12:00PM, conceptual questions on the entirety of the material, similar to what is to be expected on the SciComp comprehensive examination.

Test	Date	Questions	Solutions
1	01/28	test1.pdf	sol1.pdf
2	04/04	test2.pdf	sol2.pdf
3	04/29	test3.pdf	sol3.pdf

Course materials

Course topics

MAT. Matrix operations.

FAC. Matrix factorizations.

CND. Conditioning and stability

BAS. Change of basis, aka linear systems

EIG. Eigenvalues.

ITR. Iterative methods

DNN. Deep neural networks

Textbook

Numerical Linear Algebra, by L.N. Trefethen and D. Bau.

Bibliography

Linear algebra

Matrix Computations, by G.H. Golub and C.F. Van Loan

Applied Numerical Linear Algebra, by J.W. Demmel

Matrix Iterative Analysis, by R.S. Varga

Classic mathematics texts with strong links to linear algebra applications

Methods of Mathematical Physics, by R. Courant and D. Hilbert

Methods of Theoretical Physics, by P.M. Morse and H. Feshbach

Mathematics for the Physical Sciences, by L. Schwartz

Computational Functional Analysis, by R. Moore

Class slides

Class notes will be provided to summarize class discussion, and are posted on this website.

Week	Date	Topic	Tuesday	Thursday
01	01/19	MAT	Lesson01	Lesson02
02	01/26	MAT	Lesson03 Slides03	Lesson04 Slides04
03	02/02	FAC	Slides05	Lesson06 Slides06
04	02/09	FAC	Slides07	Slides08
05	02/16	FAC	UNC Wellness Day	Adverse Weather Cancellation
06	02/23	FAC	Slides09	Midterm exam 1
07	03/02	CND	Slides10	Slides11
08	03/09	CND	Slides12	Wellness Day
09	03/16	BAS	Slides13	Slides14
10	03/23	BAS	Slides15	Slides16
11	03/30	EIG	Lesson16 (Webinar16)	Lesson17 (Webinar17)
12	04/06	ITR	Lesson18 (Webinar18)	Midterm exam 2
13	04/13	ITR	Lesson19 (Webinar19)	FreeFEM matrices
14	04/20	ITR	JupyterLab, f2py, Ritz values	Lesson20 (Webinar20)
15	04/27	ITR	Lesson21 (Webinar21)	Midterm exam 3
16	05/04	ITR	Lesson22 (Webinar22)	

Homework

Homework generally consists of exercises from the textbook.

Nr.	Issue Date	Due Date	Topic	Problems	Solution
01	02/02	02/16	MAT	hw01.tm hw01.pdf	sol01.tm sol01.pdf
2&3	03/18	03/30	FAC	hw02.tm hw02.pdf	sol02.tm sol02.pdf
04	03/30	04/06	EIG		
05	03/26	04/23	ITR	hw05.tm hw05.pdf	
06	04/17	04/27	ITR	hw06.tm hw06.pdf	

Software

Modern software systems allow efficient, productive formulation and solution of mathematical models. A key goal of the course is to familiarize students with these capabilities, using the SciComp@UNC environment in which tools required for data analysis have been preconfigured for immediate use. Follow instructions at SciComp@UNC to install on a laptop with at least 48GB free disk space and that conforms to CCI minimal standards.

Tutorials

Software usage is introduced gradually in each class, so the first resource students should use is careful, active reading of the material posted in class. In particular, carry out small tasks until it becomes clear what the software commands accomplish. Some additional resources:

- Mathematica
  - http://www.wolfram.com/language/fast-introduction-for-math-students/en/
  - http://www.wolfram.com/wolfram-u/catalog/gen005/
  - http://www.wolfram.com/language/fast-introduction-for-programmers/en/
- TeXmacs:
  - http://www.texmacs.org/tmweb/help/tutorial.en.html

  - https://www.youtube.com/watch?v=mlcqGRv7xhc

- Julia:

  - https://julialang.org/learning/

- Scheme:

  - https://www.scheme.com/tspl4/

Course material repository

Course materials are stored in a repository that is accessed through the subversion utility, available on all major operating systems. The URL of the material is http://mitran-lab.amath.unc.edu/courses/MATH662

In the SciComp@UNC virtual machine the initial checkout can be carried out through the terminal commands

```
cd ~/courses
make MATH662
```

Update the course materials before each lecture by:

```
cd ~/courses
svn update
```

Links to course materials will also be posted to this site, but the most up-to-date version is that from the subversion repository, so carry out the svn update procedure prior to each lecture.