

$$\boldsymbol{f}_n(\boldsymbol{x})=\boldsymbol{A}_n\boldsymbol{x},\boldsymbol{f}_2(\boldsymbol{x})=\boldsymbol{A}_2\boldsymbol{x},\boldsymbol{f}_1(\boldsymbol{x})=\boldsymbol{A}_1\boldsymbol{x}$$

$$(\boldsymbol{f}_n\circ\ldots\circ\boldsymbol{f}_2\circ\boldsymbol{f}_1)(\boldsymbol{x})=\boldsymbol{A}_n\ldots\boldsymbol{A}_2\boldsymbol{A}_1\boldsymbol{x}=\boldsymbol{A}\boldsymbol{x}$$

$$(\boldsymbol{f}_n\circ\ldots\circ\boldsymbol{f}_2\circ\boldsymbol{\sigma}\circ\boldsymbol{f}_1)(\boldsymbol{x})=\text{DNN}(\boldsymbol{x})$$

$$\boldsymbol{A}_1=[\begin{array}{cc} \boldsymbol{f}_1(\boldsymbol{e}_1) & \boldsymbol{f}_1(\boldsymbol{e}_m) \end{array}]$$

$$\boldsymbol{A}=[\begin{array}{ccc} \boldsymbol{a}_1 & \ldots & \boldsymbol{a}_n \end{array}]=\boldsymbol{Q}\boldsymbol{R}=[\begin{array}{ccc} \boldsymbol{q}_1 & \ldots & \boldsymbol{q}_n \end{array}]\left[\begin{array}{cc} r_{11} & r_{12} \\ & r_{22} \end{array}\right], C(\boldsymbol{A})=C(\boldsymbol{Q})$$

$$M\ddot{\boldsymbol{y}}+D\dot{\boldsymbol{y}}+K\boldsymbol{y}=\boldsymbol{f},\boldsymbol{y},\boldsymbol{f}\in\mathbb{R}^m,m\gg1$$

$$\boldsymbol{y}=\boldsymbol{B}\boldsymbol{z},\boldsymbol{B}\in\mathbb{R}^{m\times n},n\ll m,\boldsymbol{B}^T\boldsymbol{B}=\boldsymbol{I}$$

$$\boldsymbol{B}\boldsymbol{B}^T(\boldsymbol{M}\boldsymbol{B}\ddot{\boldsymbol{z}}+\boldsymbol{D}\boldsymbol{B}\dot{\boldsymbol{z}}+\boldsymbol{K}\boldsymbol{B}\boldsymbol{z})=\boldsymbol{B}\boldsymbol{B}^T\boldsymbol{f}\Rightarrow$$

$$\boldsymbol{B}^T(\boldsymbol{M}\boldsymbol{B}\ddot{\boldsymbol{z}}+\boldsymbol{D}\boldsymbol{B}\dot{\boldsymbol{z}}+\boldsymbol{K}\boldsymbol{B}\boldsymbol{z})=\boldsymbol{B}^T\boldsymbol{f}\Rightarrow$$

$$\boldsymbol{B}^T\boldsymbol{M}\boldsymbol{B}\ddot{\boldsymbol{z}}+\boldsymbol{B}^T\boldsymbol{D}\boldsymbol{B}\dot{\boldsymbol{z}}+\boldsymbol{B}^T\boldsymbol{K}\boldsymbol{B}\boldsymbol{z}=\boldsymbol{B}^T\boldsymbol{f}$$